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Tax Smoothing with Stochastic Interest Rates: A Reassessment of Clinton's Fiscal Legacy

The return to "sound" fiscal policy after the high budget deficits of the 1980s and early 1990s has been hailed by many as the Clinton administration's most important achievement. We evaluate post-war, U.S. fiscal policy using a generalized tax-smoothing model that allows for stochastic interest rates and growth rates. We show that contrary to conventional wisdom, the evolution of the U.S. debt-GDP ratio during the 1980s was remarkably consistent with the tax-smoothing paradigm. In fact, a more substantial departure occurred during the late 1990s, when the debt-GDP ratio fell more rapidly than predicted by optimal tax smoothing.

JEL codes: E4, E6, H6

Keywords: U.S. budget deficits, tax smoothing, stochastic interest rates.

IN A SEMINAL PAPER, Barro (1979) developed a positive theory of debt determination, which generated the classic tax-smoothing result and implications for the evolution of the public debt. He demonstrated that between 1916 and 1976, government debt policy in the UK and the U.S. was surprisingly consistent with his simple theory. Recently, however, many have argued that the debt experiences of the U.S. (and other OECD economies) in the 1980s were seriously at odds with the predictions of the tax-smoothing paradigm.¹ The basic theory implies that the budget deficit should only increase temporarily in response to shocks to

1. See, among others, Roubini and Sachs (1989), Alesina and Tabellini (1990), and Alesina and Perotti (1995).

The authors thank participants at seminars at Toronto, Queen's, the 2000 Canadian Macroeconomics Study Group meetings, and the 2001 Canadian Economics Association meetings. Funding from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

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Received February 15, 2002; and accepted in revised form May 19, 2003.

Journal of Money, Credit, and Banking, Vol. 37, No. 4 (August 2005)

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government spending and growth, whereas the budget deficits in the 1980s and early 1990s were persistently high (see Figure 1). In a recent assessment of U.S. fiscal policy, Alesina (2000) states: “While the mediocre growth performance in the period 1979–82 contributes to the increase in deficits, the rest of the 1980s clearly show a radical departure from tax smoothing, as budget deficits accumulated in a period of peace and sustained growth.” He concludes that “the fiscal policy of the 1980s was unsound from the point of view of tax smoothing.”²

In 1993, perhaps heeding economists’ criticisms, the U.S. congress passed the Omnibus Budget Reconciliation Act that included a variety of tax increases and spending cuts. Although income tax rates for low-income individuals were not affected, those for high-income earners were increased, as were the corporate tax rates.³ As Figure 1 illustrates, this policy measure along with strong GDP growth contributed to dramatic reductions in budget deficits and debt in the late 1990s. The reduction in the public debt has been widely hailed in many corners and is viewed as a major achievement of the Clinton administration.

In this article, we argue that the high budget deficits and rising public debt in the 1980s were caused mainly by shocks to the interest rate and GDP growth rate, rather than any significant departure from sound fiscal policy. Taking these shocks into account, we show that U.S. fiscal policy in the 1980s was perfectly consistent

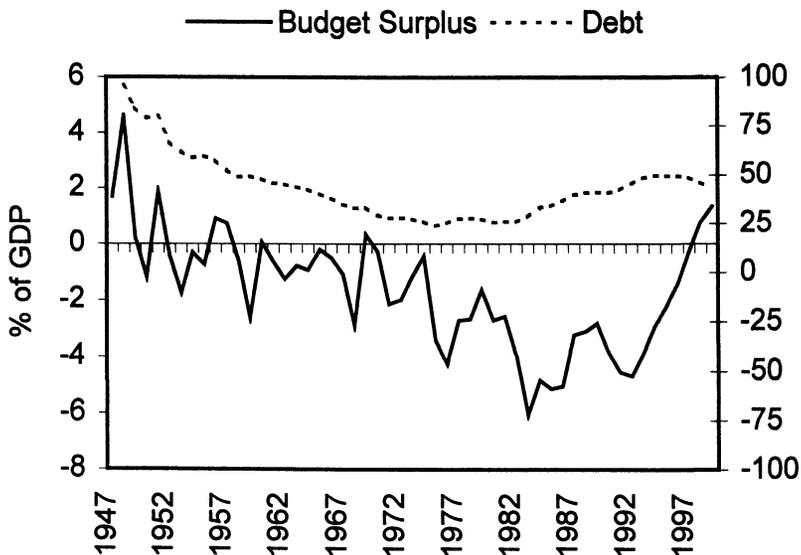


FIG. 1. U.S. budget surplus and debt

2. Underlines added by the authors.

3. A new 36% bracket was introduced for individual taxable income in excess of \$115,000 and for joint taxable income in excess of \$140,000. A new 39.6% bracket on income over \$250,000 was also introduced. Corporate taxpayers with incomes in excess of \$10 million were moved to a marginal tax rate of 35% (rather than 15% or 34%).

with tax smoothing. Rather, we contend that it is the recent budget cuts and the rapid reduction of the U.S. public debt that represent a more significant departure from the principle of tax smoothing.

Figure 2 illustrates the primary deficit along with the overall budget deficit. While, on average, the primary deficits in the late 1970s and early 1980s were higher than those in the period before 1975, this was mainly because of two drastic but *temporary* increases in the primary deficit during the two big recessions: 1974–76 and 1981–83. The reason that the budget deficits were *persistently* high is that interest payments on the debt as a percentage of GDP increased significantly during those years. As Figure 3 illustrates, the growth-adjusted interest rate switched from being negative to positive during the 1980s, mainly due to high interest rates. Figure 4 illustrates how the debt–GDP ratio would have evolved if the growth-adjusted effective interest rate⁴ had remained at a constant level equal to the pre-1980 average: *there would have been no increase in debt–GDP ratio in the 1980s*. It is obvious from this counter-factual that the high interest rates and low growth rates of the late 1970s and early 1980s largely account for the rising debt–GDP ratio.⁵

Should the Reagan and Bush administrations have significantly raised the tax rate to offset the impact of rising interest rates on the debt? What is the optimal tax response to interest and growth rate shocks implied by the tax-smoothing theory? Barro’s (1979) model implies that the average effective tax rate should be given by

$$\tau_t = \hat{r}b_t + g_t^P, \tag{1}$$

where \hat{r} denotes the growth-adjusted effective interest rate, b_t denotes the debt–GDP ratio, and g_t^P is the permanent component of the spending–GDP ratio (Roubini and Sachs 1989).⁶ Thus, the optimal tax rate is set so as to cover the interest payments on the debt and the spending that are expected to be permanent. As noted above, the sustained increase in the debt during the 1980s was largely due to a significant increase in \hat{r} rather than a significant increase in g_t^P . The view that recent experience is not consistent with tax smoothing is based on the argument that the rise in $\hat{r}b_t$ should have resulted in a one-for-one increase in the effective tax rate. However, Barro’s basic model cannot be directly used to address these questions because it assumes deterministic interest and growth rates, so that any movement in \hat{r} is effectively viewed as permanent. If the policy maker takes into account the potential variation in \hat{r} , the optimal tax rate is not given by Equation (1).

In this paper, we generalize Barro’s tax-smoothing model to allow for stochastic variation in \hat{r} and use it to assess the usefulness of the tax-smoothing theory in accounting for post-war U.S. fiscal policy. We characterize the optimal tax policy in this model and show that the average tax rate should not rise one-for-one with

4. The growth-adjusted effective interest rate equals the average nominal interest rate the federal government pays on its debt minus the nominal GDP growth rate.

5. The interest rate and GDP growth rate also played important roles prior to 1975. Despite budget deficits for most of the years between 1955 and 1974, the debt-to-GDP ratio declined sharply because the interest rate on debt was significantly below the GDP growth rate.

6. If spending were i.i.d., g_t^P would be a constant.

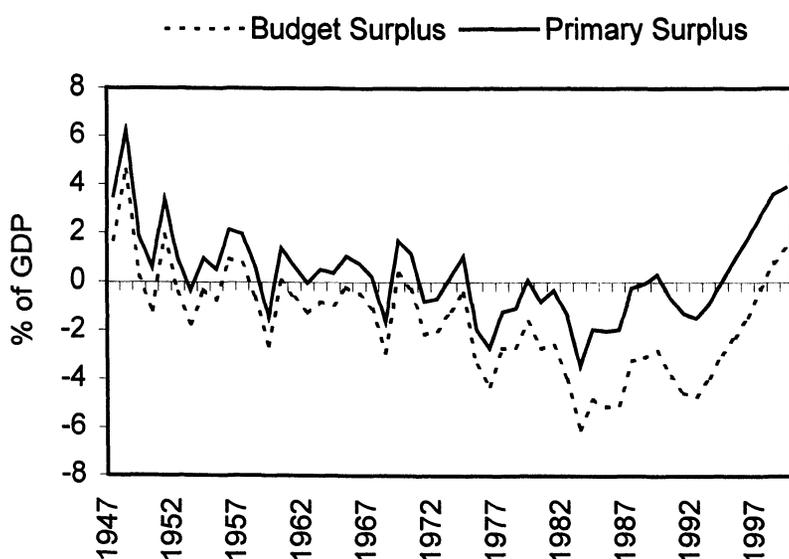


FIG. 2. Budget surplus and primary surplus

the interest payments on the debt. In fact, the optimal tax response to an increase in debt due to an interest rate shock should be very modest—of the same order of magnitude as the response to a transitory spending shock. The intuition for this follows directly from the basic principles of tax smoothing: an increase in the growth-adjusted interest rate is like a pure transitory increase in government expenditures in that it increases the stock of government debt as a percentage of GDP with no direct impact on future government expenditures. The optimal tax response, then, is to have a small but permanent increase in the tax rate that will pay off the increase in the stock of debt gradually over time.

When we calibrate the parameters of our model to match post-war U.S. data, we find that the optimal marginal response of taxes to both the debt and temporary government spending shocks is quantitatively small, and that the dynamics of the surplus and debt-to-GDP ratios implied by the tax-smoothing theory match the actual data remarkably well. Indeed, under our benchmark calibration, the departure of the surplus from the tax-smoothing model was more substantial during the late 1990s than it was during the 1980s. More generally, for reasonable parameter values, we find no evidence that the persistent budget deficits generated by the Reagan administration were inconsistent with tax smoothing.⁷

Throughout our analysis, we (like Barro) take the interest rate faced by the government to be independent of fiscal policy. There are several reasons to believe that an exogenous interest rate process may not be a bad assumption empirically. The

7. Relatedly, Ball, Elmendorf, and Mankiw (1998) argue that since with high probability the U.S. growth rate exceeds the ex-post interest rate, a large but temporary deficit may be welfare improving.

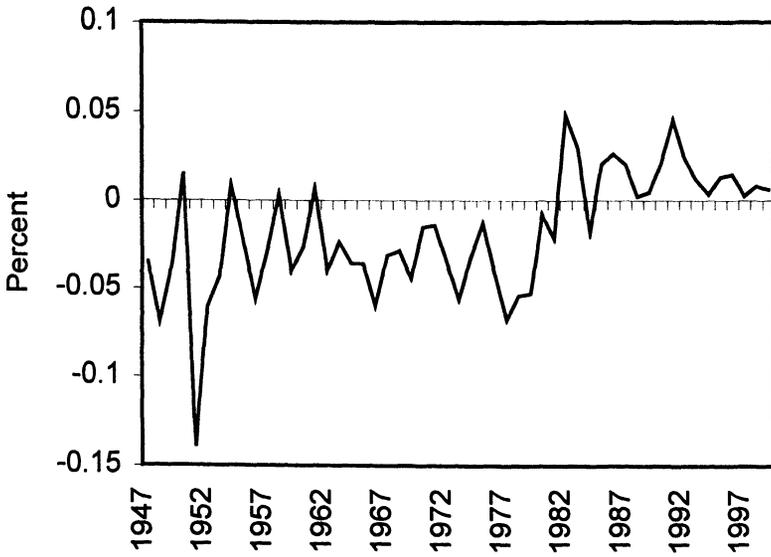


FIG. 3. Growth-adjusted interest rate

evidence of Huizinga and Mishkin (1986) and Clarida, Gali, and Gertler (2000) suggests that the high interest rates in the 1980s were mainly caused by a regime change in monetary policy in the late 1970s and early 1980s, and Plosser (1982) and Evans (1987) find at most a small effect of budget deficits on interest rates.

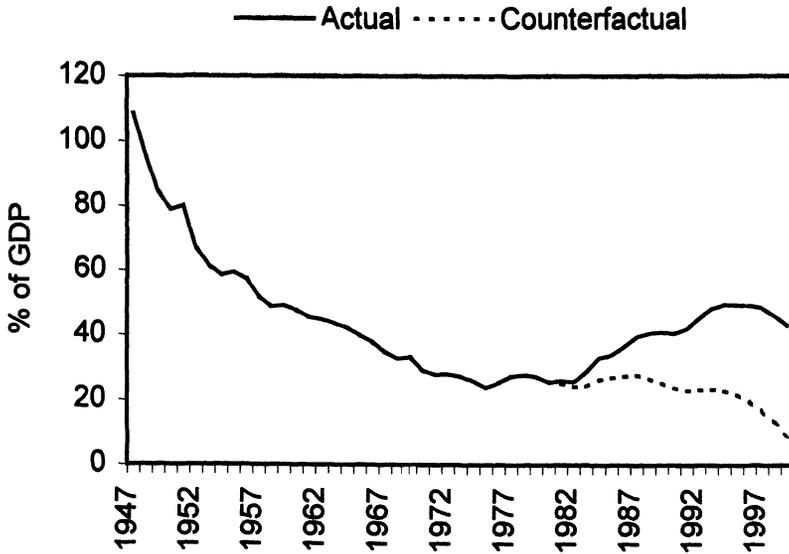


FIG. 4. Interest rate shocks and the debt

Moreover, since the main purpose of this paper is to study the quantitative response to interest rate shocks, we need a model that generates a realistic distribution of such shocks. Standard equilibrium business cycle models have difficulties in generating a realistic interest rate process. To mimic the interest rate movements in the data, we would still have to introduce some exogenous interest rate shocks in a general equilibrium model.

In their general equilibrium analyses of optimal taxation, Lucas and Stokey (1983), Zhu (1992), and Chari, Christiano, and Kehoe (1994) all assume that the government uses state-contingent debt. While the degree of insurance that the government actually enjoys is unclear, the extent to which these models are consistent with the data is controversial. In particular, they have the strong implication that the debt-GDP ratio decreases during periods when government expenditures are temporarily high and increases when government expenditures are temporarily low, purely because of the state contingency. Moreover, as pointed out by Aiyagari et al. (2002), the persistence of the optimal debt-GDP ratio implied by these models is significantly lower than is observed in the data. Imposing the restriction that the government can only issue risk-free debt may generate more realistic debt dynamics. However, analyzing the optimal taxation problem in a general equilibrium model with risk-free borrowing is computationally very difficult (Chari, Christiano, and Kehoe 1995, p. 366).⁸

Recently, Angeletos (2002) has shown that, in theory, the optimal tax policy under state-contingent debt can be replicated using risk-free borrowing if the government structures the maturity of its debt optimally in the face of shocks. However, as illustrated by Marcet and Scott (2000) and Lloyd-Ellis and Zhu (2001), there is little evidence that the U.S. or other governments follow such policies—there appears to be plenty of room for further risk management by adjusting the maturity structure or otherwise. Moreover, Buera and Nicolini (2001) find that the debt positions needed to sustain the optimal allocation in a model like that of Angeletos (2002) are unrealistically high (on the order of a few hundred times GDP).⁹ Our objective in this paper is not to develop a normative model of how U.S. fiscal policy should be carried out. Rather it is to demonstrate that the tax-smoothing paradigm (appropriately generalized) cannot be rejected as a positive explanation of U.S. debt policy, based on the growth of the debt-GDP ratio in the 1980s.

The rest of the paper is organized as follows: Section 1 develops the model and Section 2 characterizes the optimal tax policy under stochastic interest rates. Section 3 provides several analytically tractable examples to illustrate the main qualitative implications of the model. Section 4 studies the quantitative implications for the debt-GDP ratio that result when the model is calibrated to U.S. data, and Section 5 provides some concluding remarks. Technical details are relegated to the Appendix.

8. Aiyagari et al. (2002) are the only one that we know of who tackle such a problem. But they do not consider interest rate shocks.

9. Note, however, that if the government can invest in short-term assets whose returns are highly responsive to output or expenditure shocks, the optimal debt positions can be much lower (Angeletos 2002, Appendix B).

1. THE MODEL

We extend Barro's (1979) tax-smoothing model by allowing for stochastic interest and GDP growth rates. In this model, output, interest rate, and government expenditures are taken as exogenous, and the government can finance its expenditures through taxation or by issuing nominally risk-free debt. Throughout the paper, the interest and GDP growth rates to which we refer are always nominal.

Let Y_t denote GDP, P_t the price level, G_t government expenditures, and τ_t the tax rate in period t . Let B_t be the stock of public debt at the beginning of period t and r_{t-1} the (continuously compounding) risk-free nominal interest rate paid on the debt in period t ,¹⁰ which is determined in period $t - 1$. Normalizing gives us the debt-GDP ratio, $b_t = B_t / P_{t-1} Y_{t-1}$, expenditure-GDP ratio $g_t = G_t / P_t Y_t$, and the growth rate of nominal GDP $v_t = \ln(P_t Y_t / P_{t-1} Y_{t-1})$. The government's period-by-period budget constraint can therefore be expressed in GDP units as

$$b_{t+1} = \exp(r_{t-1} - v_t) b_t + g_t - \tau_t. \quad (2)$$

Taxes impose a deadweight loss on the economy in period t that is proportional to GDP and a quadratic function of the tax rate

$$\frac{1}{2} \tau_t^2 Y_t. \quad (3)$$

The government's objective is to choose the optimal tax policy that minimizes the present discounted expected deadweight losses¹¹

$$V(b_0) = \max_{\{\tau_t\}_{t \geq 0}} - \frac{1}{M_0} \sum_{t=0}^{\infty} E_0 M_t \frac{1}{2} \tau_t^2 Y_t, \quad (4)$$

subject to the flow budget constraint (Equation 2) and the no-Ponzi game restriction

$$\lim_{j \rightarrow \infty} E_t M_{t+j} b_{t+j+1} Y_{t+j} \leq 0. \quad (5)$$

Here, we assume that the government uses the market stochastic discount factor, M_t , to discount future deadweight losses. For the post-war period, the average nominal GDP growth rate exceeded the average nominal one-year interest rate. If we use the average one-year interest rate as the discount rate for the government, the government's objective function would be unbounded and the optimal policy would not be well-defined. However, with a stochastic discount factor, this is not a problem provided that the risk premium associated with GDP growth shocks is sufficiently large.¹² In addition, when tax rates, interest rates, and GDP growth

10. Let r' be the ratio of interest payments to debt. We define the effective interest rate as $r = \ln(1 + r')$, so that the gross interest is e^r . This transformation is for analytical convenience only.

11. This is the same assumption used by Barro (1979).

12. In the literature, authors have side-stepped this problem by using the interest rate on long-term bonds rather than one-year interest rate on debt. But there is no justification for using a long-term interest rate to discount annually.

rates are deterministic, discounting using the stochastic discount factor is equivalent to discounting using one-year interest rate.

Standard arguments can be used to show that the government's optimal tax policy is characterized by the following first-order condition:

$$\tau_t Y_t = E_t \left[\frac{M_{t+1}}{M_t} \exp(r_t - v_{t+1}) \tau_{t+1} Y_{t+1} \right], \quad (6)$$

and the transversality condition (Equation 5). If we define the nominal stochastic discount factor as

$$M_t^P = M_t / P_t, \quad (7)$$

then the first-order condition (Equation 6) can be rewritten more succinctly as

$$\tau_t = E_t [q_{t+1} \tau_{t+1}], \quad (8)$$

where

$$q_{t+1} = \frac{M_{t+1}^P}{M_t^P} \exp(r_t). \quad (9)$$

Since r_t is the risk-free nominal interest rate, the no-arbitrage condition implies that

$$E_t [q_{t+1}] = E_t \left[\frac{M_{t+1}^P}{M_t^P} \exp(r_t) \right] = 1. \quad (10)$$

Let z_t represent a vector of exogenous shocks in period t , which include r_t , v_t , g_t , and any shocks to M_t^P , and let $z^{(t)}$ be the history of the shocks up to t . Assume that $z^{(t)}$ has a well-defined probability density function $\pi_t(z^{(t)})$. Then, Equation (10) implies that

$$\hat{\pi}_t(z_{t+1} | z^{(t)}) = q_{t+1} \pi_t(z_{t+1} | z^{(t)}) \quad (11)$$

is also a conditional density function, which we call the risk-adjusted probability density function. Under this risk-adjusted probability density function, Equation (8) can be written as

$$\tau_t = \hat{E}_t [\tau_{t+1}]. \quad (12)$$

PROPOSITION 1: *The optimal tax rate follows a martingale process under the risk-adjusted probability distribution.*

If both the interest rate r_t and the growth rate v_t are constant, and the government uses the interest rate as its discount rate, then, $q_{t+1} \equiv 1$, and we have Barro's tax-smoothing result that the optimal tax rate follows a martingale process under the original probability distribution. Proposition 1 is simply a generalization of Barro's result to the case of a stochastic interest rate and a stochastic GDP growth rate. The key implication of Barro's model, that the tax rate follows a martingale process,

remains valid in the generalized model under the risk-adjusted probability distribution. In the next section, we turn to characterizing the optimal tax policy in the presence of shocks to interest rate, GDP growth rate, and government expenditures.

2. CHARACTERIZING THE OPTIMAL TAX POLICY

When nominal interest and GDP growth rates are constants, the optimal tax policy has a very simple representation given in Equation (1), where $\hat{r} = e^{r-v} - 1$. That is, the optimal tax rate is set so as to cover the debt-servicing requirement and the permanent component of government expenditure. However, with stochastic variation in interest and growth rates, we cannot simply replace r and v in Equation (1) with their stochastic counterparts, because the optimal policy should take this variation into account. In this section, we specify more explicitly the shock processes and the stochastic discount factor underlying the optimization problem described above. We show that under these specifications, the optimal tax policy turns out to have a representation similar to Equation (1), but that the sensitivity of taxes to changes in the debt-service component depends on the persistence of these changes.¹³

The stochastic discount factor: we directly specify a parametric process for the stochastic discount factor

$$-\ln\left(\frac{M_{t+1}^P}{M_t^P}\right) = r_t + \frac{1}{2}\sigma_m^2 + \varepsilon_{m,t+1}, \tag{13}$$

where $\varepsilon_{m,t+1}$ is an i.i.d. variable with distribution $N(0, \sigma_m^2)$. This specification ensures that the no-arbitrage condition (Equation 10) for the nominal interest rate is always satisfied. This approach has recently been used by several authors to study the term-structure of interest rates and to analyze the optimal portfolio allocation problem.¹⁴ It has the advantage of being able to generate realistic distributions of interest rates and asset returns, which is important for our analysis of optimal policy under stochastic interest rates.

For any risky nominal return $r_{i,t+1}$, the following no-arbitrage condition must hold

$$E_t\left[\frac{M_{t+1}^P}{M_t^P} \exp(r_{i,t+1})\right] = 1. \tag{14}$$

If we assume that the unexpected return $\varepsilon_{i,t+1} = r_{i,t+1} - E_t[r_{i,t+1}]$ has a normal conditional distribution, then, substituting Equation (13) into Equation (14) implies that

13. The nature of the optimal tax policy remains the same under much more general specifications than those considered here. We adopt these particular specifications to facilitate the quantitative analysis of Section 4.

14. See, for example, Campbell, Lo, and MacKinlay (1997) and Campbell and Viceira (2001).

$$E_t[r_{i,t+1} - r_t] + \frac{1}{2} \text{Var}_t(\varepsilon_{i,t+1}) = \text{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{m,t+1}). \quad (15)$$

That is, the expected excess return of asset i (after adjusting a variance term for log-returns) equals the conditional covariance between the asset return and the innovation in the stochastic discount factor, which measures the risk premium on asset i . We assume that $\varepsilon_{m,t+1}$ is proportional to the unexpected return of the market portfolio. So, our model implies that the expected excess return to asset i equals the conditional covariance between the asset return and the unexpected return of the market portfolio, which is the same implication of the standard capital asset pricing model (CAPM).

The shock processes: the interest rate is assumed to follow a first-order Markov process, and the processes for GDP growth rates and government expenditures are given by the following equations:

$$v_{t+1} = \bar{v} + \frac{1}{2} \sigma_v^2 + \varepsilon_{v,t+1}, \quad (16)$$

$$g_{t+1} = (1 - \rho_g)\bar{g} + \rho_g g_t + \varepsilon_{g,t+1}, \quad (17)$$

where $0 < \rho_g < 1$, and $\varepsilon_{v,t+1}$ and $\varepsilon_{g,t+1}$ are independent i.i.d. variables with distributions $N(0, \sigma_v^2)$ and $N(0, \sigma_g^2)$, respectively. We further assume that $\{r_t\}_{t \geq 0}$ is independent of $\{\varepsilon_{v,t}\}_{t \geq 0}$ and $\{\varepsilon_{g,t}\}_{t \geq 0}$.¹⁵

Growth-risk premium: we assume that innovations to the stochastic discount factor $\{\varepsilon_{m,t}\}_{t \geq 0}$ are independent of $\{r_t\}_{t \geq 0}$ and government expenditure shocks $\{\varepsilon_{g,t}\}_{t \geq 0}$, but are correlated with shocks to GDP growth $\{\varepsilon_{v,t}\}_{t \geq 0}$. We also assume that the random vector $(\varepsilon_{m,t}, \varepsilon_{v,t})$ is i.i.d. and has a joint normal distribution with a constant covariance given by

$$\gamma = \text{Cov}(\varepsilon_{v,t}, \varepsilon_{m,t}). \quad (18)$$

From Equation (15) we know that γ may be interpreted as the risk premium associated with shocks to the GDP growth rate.

Given these assumptions, a fairly straightforward characterization of the optimal tax policy is possible.

PROPOSITION 2: *If there exists a function $\phi(\cdot)$ and a constant $\phi^* > 0$, such that $0 < \phi^* \leq \phi(r_t) < 1$ and*

$$\phi(r_t) = \frac{e^{r_t - \bar{v} + \gamma} E_t[\phi(r_{t+1})]}{1 + e^{r_t - \bar{v} + \gamma} E_t[\phi(r_{t+1})]}, \quad (19)$$

then, the optimal tax rate is given by

15. In our 1955–99 sample data, these correlations turn out to be very small (both contemporaneous and lagged), and ignoring them greatly simplifies both the theoretical and quantitative analyses. However, we consider the qualitative implications of allowing for them in Section 4.3.

$$\tau_t = \phi(r_t)e^{r_{t-1}-v_t} b_t + g_t^p, \tag{20}$$

where

$$g_t^p = \bar{g} + \psi(r_t)(g_t - \bar{g}), \tag{21}$$

and $\psi(r_t) < 1$ is the unique bounded solution to the linear functional equation

$$\psi(r_t) = (1 - \phi(r_t))\rho_g E_t[\psi(r_{t+1})] + \phi(r_t). \tag{22}$$

PROOF: See Appendix.

In the Appendix, we show that a function $\phi(\cdot)$ satisfying the conditions in Proposition 2 exists provided that

$$r_t - \bar{v} + \gamma \geq \delta \text{ almost surely} \tag{23}$$

for some constant $\delta > 0$. Condition (23) is non-trivial. In the U.S., the interest rate during the post-war period was often below the average GDP growth rate. For Condition (23) to hold, the risk premium on shocks to the growth rate, γ , must be sufficiently large. Condition (23) is only a sufficient condition. Even for a value of γ , such that the condition does not hold, there may still exist a uniformly bounded solution to Equation (19).

Thus, if the risk premium is sufficiently large, Proposition 2 implies that the optimal tax rate has a similar representation as that when interest and growth rates are constants. In particular, it shows that the optimal tax rate can be decomposed into two parts: the tax response to debt, $\phi(r_t) \exp(r_{t-1} - v_t)b_t$, and permanent government expenditure, g_t^p , which is the sum of the long-term mean of government expenditures, \bar{g} , and the permanent component of government expenditure shocks, $\psi(r_t)(g_t - \bar{g})$. Shocks to the interest rate and the GDP growth rate affect the optimal tax policy through their impacts on the debt–GDP ratio and through their impacts on the marginal responses of the tax rate to debt and government expenditure shocks.

Let $\theta_t = \phi(r_t) \exp(r_{t-1} - v_t)$ denote the marginal tax response to the debt, and let $\hat{r}_t = e^{r_t - v_t} - 1$ denote the growth-adjusted interest rate on the debt. Then, the evolution of the debt–GDP ratio implied by the optimal tax policy is described by

$$b_{t+1} - b_t = [1 - \psi(r_t)](g_t - \bar{g}) + (\hat{r}_t - \theta_t)b_t. \tag{24}$$

Since $\psi(r_t) < 1$, the debt–GDP ratio will increase if there is a positive shock to government expenditures. Starting from a positive level, the debt–GDP ratio will also increase in the absence of government expenditure shocks whenever the marginal tax response to debt, θ_t , is less than the growth-adjusted interest rate, \hat{r}_t . Note that if there were no interest or growth shocks, then in effect $\hat{r}_t = \theta_t$.

3. SOME ILLUSTRATIVE EXAMPLES

In Section 4, we numerically characterize the quantitative implications of our tax-smoothing model calibrated to U.S. data. However, in order to develop some

intuition for the nature of our results, it is useful to consider a number of special cases that are solved analytically in the Appendix.

Example 1: Holding the interest rate constant, whenever the GDP growth rate is far enough below its average level, the optimal debt–GDP ratio rises, even in the absence of government expenditure shocks.

This case is almost identical to Barro’s original model except that nominal GDP growth is stochastic, and the implied optimal tax policy is the same, once we replace the interest rate \bar{r} with its risk-adjusted counterpart $\bar{r} + \gamma$. In the absence of spending shocks, the optimal growth in the debt–GDP ratio is then given by

$$\hat{r}_t - \theta_t = e^{\bar{v} - v_t - \gamma} - 1. \quad (25)$$

Since $\gamma > 0$, the marginal tax response to debt, θ_t , exceeds the effective interest rate, \hat{r}_t , on average and, as a result, the optimal debt–GDP ratio declines on average in the absence of shocks to government expenditure (as a percentage of GDP). However, whenever the realized GDP growth rate is lower than average so that $v_t < \bar{v} - \gamma$, then $\theta_t < \hat{r}_t$ and the optimal debt–GDP ratio grows.

Example 2: With zero persistence in government spending, the optimal marginal tax response to the gross debt–GDP ratio is of the same order of magnitude as would be the response to transitory spending shocks.

If $\rho_g = 0$, the optimal tax policy is given by

$$\tau_t = \bar{g} + \phi(r_t)[\exp(r_{t-1} - v_t)b_t + g_t - \bar{g}]. \quad (26)$$

Thus, in this case, the marginal tax response to the gross debt–GDP ratio, $\exp(r_{t-1} - v_t)b_t$, is *identical* to the marginal response to pure transitory shocks to government expenditures.¹⁶ The optimal tax response to purely transitory expenditure shocks is relatively small—indeed, the key idea of tax smoothing is that the tax should not fully respond to non-permanent increases in spending. This example, therefore, implies that we should not expect a significant increase in the optimal tax rate due to a rise in public debt caused by an increase in the growth-adjusted interest rate. The intuition behind this result is as follows: an increase in the growth-adjusted interest rate is like a pure transitory increase in government expenditures in that it increases the stock of government debt as a percentage of GDP with no direct impact on future government expenditures. The optimal tax response, then, is to have a small but permanent increase in the tax rate that will pay off the increase in the stock of debt gradually over time.

Example 3: Holding nominal GDP and government expenditure constant, as long as interest rate shocks are not permanent, the optimal debt–GDP ratio rises whenever interest rates are higher than average.

In this example, we consider a two-state Markov process for the interest rate, in which the transition to the high interest rate state is not permanent. In this case, tax

16. Increasing ρ_g raises the responsiveness of the tax rate to spending shocks, but has no effect on its responsiveness to the gross debt–GDP ratio. Thus, in general, the responsiveness to the gross debt–GDP ratio is *less than* that to spending shocks.

smoothing implies that the *optimal* debt–GDP ratio declines in the low interest rate state and is non-decreasing in the high interest rate state. If the economy were to remain in the high interest rate state permanently, then the debt–GDP ratio will be a constant. If the economy remains in the high interest rate state only temporarily, then the debt–GDP ratio rises when the interest rate is high. Therefore, the optimal tax response to a positive interest rate shock depends on the persistence of the shock. If the shock is permanent, then the optimal tax response is to fully respond to the shock so that the debt–GDP ratio stays constant. If the shock is temporary, however, the optimal tax response is such that the debt–GDP ratio increases, since it is expected that the interest rate will decline and therefore debt–GDP ratio will decline in the future.

Example 4: Even if interest rate increases are expected to be permanent, the optimal debt–GDP ratio still rises during periods of lower than average GDP growth.

Suppose that everything is the same as in Example 3, except that v_t is an i.i.d. variable. If an increase in the interest rate is permanent, then the growth in the debt–GDP ratio is given by

$$\hat{r}_t - \theta_t = \exp(\bar{v} - v_t) - 1, \tag{27}$$

which is positive if $v_t < \bar{v}$, in which case the debt–GDP ratio increases (assuming $b_t > 0$). Thus, even if there is a permanent positive shock to the interest rate, the debt–GDP ratio still increases if the GDP growth rate is temporarily low. Since shocks to the GDP growth rate are generally not persistent, the optimal tax response to negative shocks to GDP growth rate is such that the debt–GDP ratio increases.

From these examples, we can see why the tax-smoothing policy implies that the budget would persistently be in surplus prior to the 1980s and persistently in deficit during the 1980s. Prior to the 1980s, the real interest rate was low and the GDP growth rate was high, so that the growth-adjusted interest rate was well below its long-term average. In this period, the optimal marginal tax response to debt should be higher than the growth-adjusted interest rate on debt, which implies that the debt–GDP ratio should decline. In the 1980s, the real interest rate increased significantly and the GDP growth rate dropped. These shocks to the interest rate and the GDP growth rate pushed the growth-adjusted interest rate above its long-term average and, in this period, the tax response to the debt should optimally be less than the growth-adjusted interest rate on debt. This, along with the temporary shocks to government expenditure, implies that the debt–GDP ratio should have optimally increased during this period. So, at least qualitatively, the dynamics of the U.S. budget appear to have been consistent with that predicted by the tax-smoothing theory.

Of course, this does not necessarily imply that the tax-smoothing model predicts fiscal deficits of the magnitude that was observed in the 1980s. To address this issue, it is necessary to compare the quantitative predictions of the model with the data.

4. QUANTITATIVE IMPLICATIONS OF TAX SMOOTHING

In this section, we study quantitatively the dynamics of the U.S. public debt implied by the optimal tax policy characterized above. To do so, we estimate

the shock processes and calibrate the risk-premium parameter γ . The data we use are described in detail in Appendix.

4.1 Estimating the Shock Processes

We assume that the interest rate also follows an AR(1) process

$$r_{t+1} = (1 - \rho_r)\bar{r} + \rho_r r_t + \varepsilon_{r,t+1}, \quad (28)$$

where $\varepsilon_{r,t}$ is an i.i.d. variable with distribution $N(0, \sigma_r^2)$. We estimate Equations (16), (17), and (28) using full information maximum likelihood. The estimated results are reported in Table 1. To solve the functional Equations (19) and (22), however, we need to discretize the process for the interest rate r_t . We do so using a ten-state Markov chain to approximate the estimated AR(1) process of r_t specified in Equation (28).¹⁷

4.2 Calibrating the Risk-Premium Parameter γ

We allow the market portfolio to consist of both financial and human capital, and approximate the return on human capital by the per capita GDP growth rate. Thus, we have

$$\varepsilon_{m,t+1} = \beta[\lambda\varepsilon_{e,t+1} + (1 - \lambda)\varepsilon_{v,t+1}], \quad (29)$$

where β is the ratio of $\varepsilon_{m,t+1}$ to the unexpected return on the market portfolio, $\varepsilon_{e,t+1}$ is the unexpected return on a market index, and λ is the weight of financial capital in the market portfolio. We assume that $\varepsilon_{e,t+1}$ is distributed normally, $N(0, \sigma_e^2)$. This specification follows that of Jagannathan and Wang (1996) who show that allowing for human capital to be part of the market portfolio can significantly improve

TABLE 1
BENCHMARK PARAMETER VALUES (1947–99)

Parameter	Estimate	Standard error
\bar{r}	0.075896	0.019886
ρ_r	0.967508	0.032462
\bar{v}	0.069598	0.029036
σ_v^2	0.000843	0.000201
σ_{ev}	0.001250	0.000298
σ_e^2	0.026773	0.006375
\bar{r}_e	0.141080	0.163630
g	0.173082	0.007984
ρ_g	0.603830	0.070695
λ	0.3	—
β	8.821584	—
γ	0.008513	—

17. The basic idea is similar to Example 3 above except that we allow ten possible values for the interest rate and calibrate the transition probabilities so that the process approximates the estimated AR process.

the fit of the CAPM in accounting for the cross-section of expected returns on the NYSE.¹⁸ They argue that aggregate loans against future human capital (e.g., mortgages, consumer credit, and personal bank loans) account for as much wealth in the U.S. as equities. Moreover, there are also active insurance markets for hedging the risk to human capital (e.g., life insurance, UI, and medical insurance). In the calibration exercise below, we not only follow Jagannathan and Wang (1996) by assuming that $\lambda = 0.3$ as the benchmark, but we also investigate the sensitivity of our results to other choices of λ .

For any given value of λ , we use the no-arbitrage condition to calibrate the value of β . From Equations (15) and (29), we have

$$E_t[r_{e,t+1} - r_t] + \frac{1}{2} \sigma_e^2 = \beta[\lambda\sigma_e^2 + (1 - \lambda)\sigma_{ev}]. \tag{30}$$

Taking unconditional expectation on both sides of the equation and solving for β yields

$$\beta = \frac{E[r_{e,t+1} - r_t] + \frac{1}{2} \sigma_e^2}{\lambda\sigma_e^2 + (1 - \lambda)\sigma_{ev}}. \tag{31}$$

Since both the variance of the unexpected market return, σ_e^2 , and the covariance between the market return and GDP growth, σ_{ev} , can be estimated from the data,¹⁹ we compute β from Equation (31) by replacing the expectation $E[r_{e,t+1} - r_t]$ with the sample mean, $\bar{r}_e - \bar{r}$. The growth-risk premium is given by

$$\gamma = \beta[\lambda\sigma_{ev} + (1 - \lambda)\sigma_v^2], \tag{32}$$

which can be computed by substituting for the value of β using Equation (31). The calibration results for the benchmark case are reported in Table 1.

Given the estimated shock processes and the calibrated parameter for the stochastic discount factor, we solve the functional Equation (19) numerically. Given the solution to Equation (19), $\phi(r_t)$, we then numerically solve the functional Equation (22) to get $\psi(r_t)$. Given $\phi(r_t)$, $\psi(r_t)$, and the initial level of the debt–GDP ratio b_0 , we calculate the optimal tax rate and the debt–GDP ratio iteratively using Equations (20) and (2). The algorithm we use to solve $\phi(r_t)$ and $\psi(r_t)$ numerically is given in the Appendix.

4.3 Results

Benchmark case. Figure 5 compares the actual tax rate to that predicted by the tax-smoothing policy for the benchmark case. In the data, we follow Barro by computing the actual effective tax rate as the ratio of tax revenues to GDP. The volatility of the predicted tax rate is somewhat less than the volatility of the actual tax rate. However, it is remarkable how well the time-average of optimal tax rate predicted by the model matches that in the data. The average level from the model

18. Jagannathan and Wang (1996) proxy the market return to human capital using the growth in labor income.

19. The details on how we estimate σ_{ve} and σ_e^2 are given in Appendix.

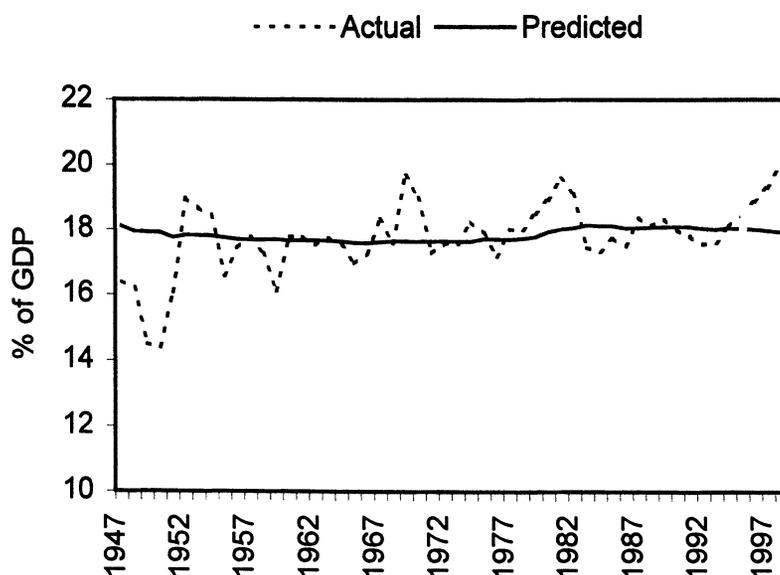


FIG. 5. Actual and predicted tax rate

is largely determined solely by the long run level of government expenditure; so this implies that, on average, during the post-war period, tax revenues have been quite consistent with intertemporal budget balance.²⁰

Despite the relative smoothness of the predicted tax rate, it can be seen from Figure 6 that the predicted budget surplus tracks the dynamics of the actual surplus well, especially during the 1980s.²¹ As a result, the evolution of the debt-GDP ratio in the benchmark case (Figure 7) is very close to that in the data. In other words, the excess volatility of the actual tax rate is neither great enough nor persistent enough to make much difference to the evolution of the debt. Given that the optimal tax is extremely smooth, it is not surprising that the implied debt-GDP ratio is very sensitive to the shocks to government expenditures and the growth-adjusted interest rate. The sharp increase in the U.S. debt-GDP ratio in the 1980s resulted from the fact that adverse interest rate and growth shocks were not offset by tax rate movements. Our results demonstrate that this is both qualitatively and quantitatively consistent with the tax-smoothing theory.

Interestingly, since 1994, however, the actual surplus-GDP ratio has been much higher than that predicted by the tax-smoothing theory, so that the debt-GDP ratio declined too rapidly. This rapid reduction in debt has been associated with a significant increase in taxes as a percentage of GDP, partly due to the new tax increases

20. This result complements the work of Bohn (1998), who finds evidence in support of the sustainability of the U.S. debt-GDP ratio.

21. Although, actual taxes appear volatile in Figure 5, this is largely a result of the scale. The impact on the overall surplus of these innovations is small.

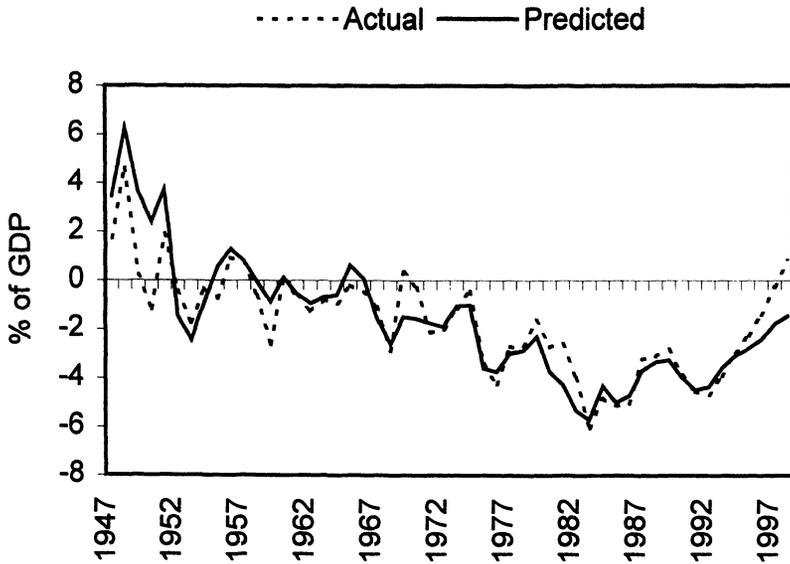


FIG. 6. Actual and predicted surplus

enacted in the 1993 Omnibus Budget Reconciliation Act. Thus, according to our benchmark calibration, the view that a major legacy of the Clinton administration was a return to “sound” fiscal policy after the “excessive” budget deficits of the 1980s seems incorrect. Rather, it suggests that the fiscal stance of the federal administration during the late 1990s was overly tight, so that the debt–GDP declined too rapidly. From Figures 5 and 6, it is obvious that there have been several other temporary departures from the optimal policy on a scale similar to that observed in the late 1990s. Other significant departures are associated with the Korean war (1949–50), the Johnson surtax (1969–70), and the early 1980s (1979–82). However, the main point that we emphasize here is that, in contrast to conventional wisdom, the departure from optimal tax smoothing was greater after 1993 (the Clinton era) than it was during the preceding decade of large budget deficits under Reagan and Bush.

Sensitivity analysis. Although, most of the parameter values we have used in the benchmark case are the maximum likelihood estimates, there is of course considerable uncertainty about their true values as quantified by the standard errors in Table 1. Moreover, our choice of $\lambda = 0.3$ for the share of financial assets in the market portfolio is somewhat arbitrary. It is, therefore, necessary to consider the sensitivity of our results to changes in the model’s parameter values.

The composition of the market portfolio, λ : although Jagannathan and Wang (1996) show that assuming that wealth consists of human and not just financial wealth improves the fit of the CAPM to U.S. market data, the appropriate value of

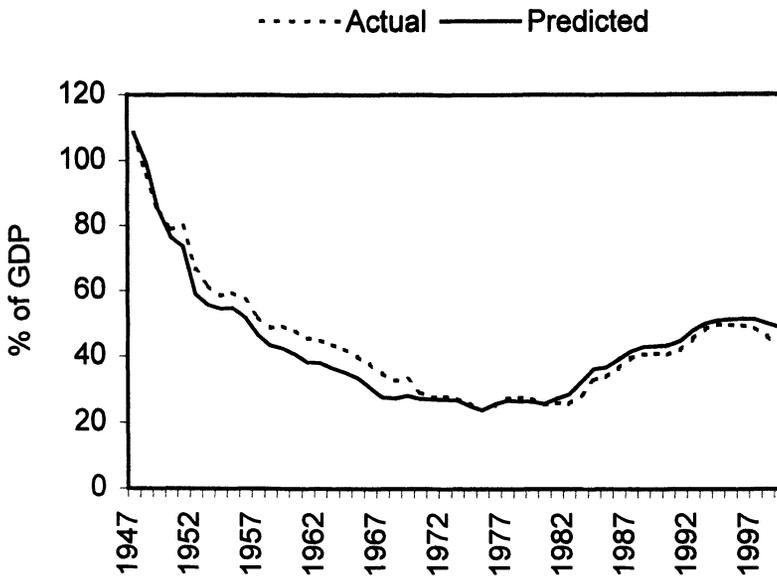


FIG. 7. Actual and predicted debt

λ is unknown.²² We, therefore, consider the sensitivity of our results to changes in the value of λ . This parameter enters the model only via the risk-premium parameter γ . Using Equations (31) and (32), it is straightforward to show that

$$\text{sign}\left[\frac{d\gamma}{d\lambda}\right] = \text{sign}[\sigma_{ev}^2 - \sigma_v^2\sigma_e^2], \quad (33)$$

and for our benchmark parameters in Table 1 it can be verified that $d\gamma/d\lambda < 0$. Thus, reducing the value of λ increases the growth-risk premium, which implies that the marginal tax response to debt is larger and the debt implied by tax smoothing is lower. Figures 8 and 9 show the surplus and the debt-GDP ratios implied by tax smoothing for $\lambda = 1, 0.3$, and 0 , respectively. The results are quantitatively very similar for $\lambda = 1$ and 0.3 . For $\lambda = 0$, the implied growth-risk premium is significantly higher and therefore the optimal tax rates are significantly higher, which implies that the predicted debt is significantly below the actual debt. However, this case represents a very extreme market portfolio consisting of no financial wealth.

The persistence of the shock processes (ρ_r, ρ_g): Example 3 shows that the optimal tax response to debt is sensitive to the persistence of interest rate shocks. In particular, a higher ρ , implies a larger marginal tax response to debt and, therefore, a smaller effect of interest rate shocks on debt. We considered the evolution of the surplus and debt-GDP ratios implied by the tax smoothing policy for the full range of values of ρ ,

22. Kandel and Stambaugh (1995) argue that even if stocks constitute a small fraction of total wealth, the stock index portfolio return could be a good proxy for the return on the portfolio of aggregate wealth.

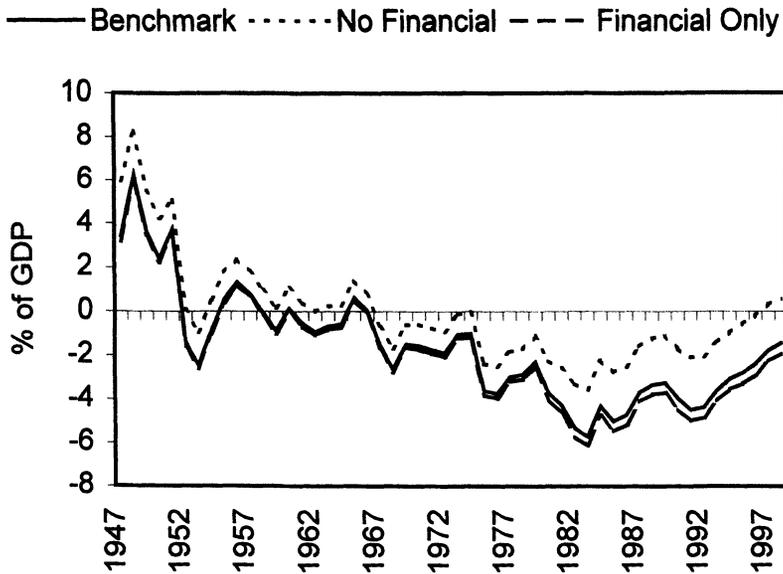


FIG. 8. Risk premium and the surplus

between 0 and 1. While it is true that the tax rates are higher for higher value of ρ_r , the quantitative difference is fairly small, and is tiny within one standard deviation of the benchmark estimate. As we demonstrated in Example 4, the marginal tax response to debt depends on the persistence of both the interest rate and the GDP growth rate. Since the GDP growth rate is i.i.d., the persistence of the growth-adjusted interest rate is quite low, even if the interest rate itself follows a random walk. As a result, the debt dynamics implied by the tax smoothing theory are not very sensitive to our assumptions regarding the persistence of interest rate shocks.

As with the interest rate process, the optimal tax policy is also largely insensitive to the persistence of shocks to government spending, ρ_g . Varying the parameter by one or two standard deviations in either direction has a quantitatively minute impact on the predicted evolution of the debt. In either case, the general dynamics of the surplus, and in particular, the persistently large budget deficits during the 1980s remain consistent with the predictions of the tax-smoothing model.

Alternative specifications of the shock processes: although our specification of the shock processes (see Section 2) allows for a correlation between innovations to the stochastic discount factor and GDP growth, we have not allowed for such a correlation between GDP growth, realized nominal interest rates, and/or government spending. In principle, we could allow for such correlations without changing the basic formulation of the optimal tax policy given in Proposition 2.²³ In our current

23. A more general version of Proposition 2, allowing for such correlations is available on request from the authors.

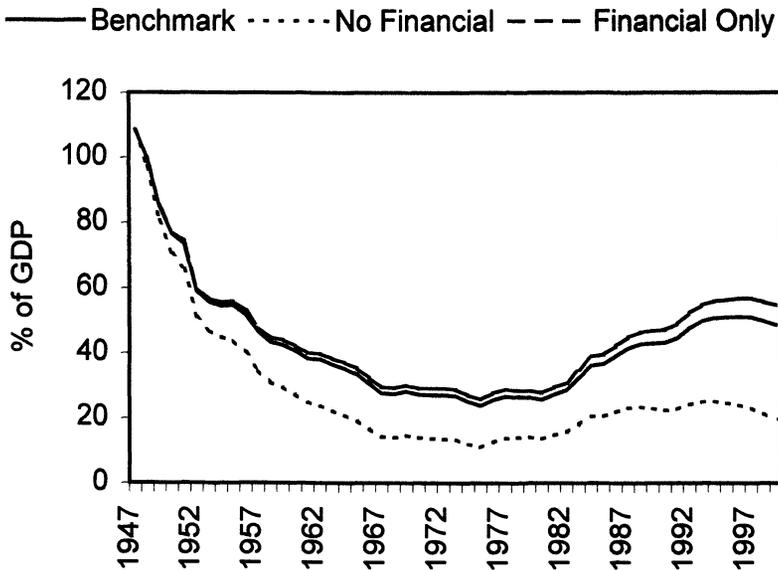


FIG. 9. Risk premium and the debt

numerical computations, calculating the optimal tax policy requires us to represent the interest rate process with a ten-state Markov process in order to approximate $\phi(r)$ and $\psi(r)$. Adding these correlations would increase the dimensionality of the state space by a factor of 3, significantly raising the computational complexity of the problem. Given that in our sample these correlations turn out to be very small—we do not find any contemporaneous or lagged correlation coefficients amongst these variables that exceed 0.1—their quantitative impact on the optimal tax rate is also going to be small.

Intuitively, the qualitative implications of adding these correlations are fairly straightforward to see. The small positive correlation between r_t and v_t would imply a smaller variance in the growth-adjusted interest rate. This, in turn, would be reflected in a smaller growth-risk premium and *even less* sensitivity of the optimal tax rate to variations in the debt–GDP ratio than we have calculated above. The small positive correlation between r_t and g_t implies that positive shocks to spending occur when they are most costly. This would cause the optimal tax rate to be more sensitive to government spending shocks, and hence somewhat less smooth over time. However, as long as these shocks are not permanent, the low sensitivity characterized above is unlikely to be affected.

5. CONCLUDING REMARKS

The movement of the U.S. public debt has been greatly influenced by variations in the interest rate and GDP growth rate. In this paper, we extend Barro's (1979)

tax-smoothing theory to allow for stochastic movements in the interest rate and the GDP growth rate. We show how the optimal response of the tax rate to increases in the debt–GDP ratio and to transitory government expenditure shocks depend on movements in the interest rate, the GDP growth rate and the risk premium associated with GDP growth variability. The optimal tax policy implies that the response to increases in the debt–GDP ratio caused by non-permanent increases in the growth-adjusted interest rate is of the same order of magnitude as the response to transitory spending shocks. As a result, during periods of higher than average interest rates and lower than average growth rates, an increase in the debt–GDP ratio takes place as part of an optimal tax-smoothing policy, even in the absence of spending shocks.

When we calibrate our model to post-war U.S. data, we find that the optimal tax rate and debt dynamics predicted by our model closely resemble those of the actual debt. In particular, we find that the sharp increases in the U.S. debt–GDP ratio in the 1980s, with no large increase in tax rates, were quite consistent with the tax-smoothing paradigm. Indeed, a more substantial departure from the principle of tax smoothing occurred during the Clinton administration when the surplus–GDP ratio rose much more rapidly than predicted by the model.

It should be recognized that the tax-smoothing paradigm is about the optimal method of financing (i.e., taxation or debt), taking as given the process for government expenditures and interest rates. Our estimated process for spending and interest rates is based on past U.S. experience. The fact that the recent debt–GDP ratio has fallen more rapidly than predicted by the model implies that taxes were too high, *given the estimated processes for spending and interest rates*. It does not necessarily imply that taxes should be cut if spending is anticipated to be persistently high in the near future. For example, if it is anticipated that the cost of social security payments will rise substantially and that this increase will be unusually persistent, then the current level of taxes may be warranted. This caveat does not, however, affect the main message of this paper: it is not possible to conclude that U.S. fiscal policy during the 1980s was unsound from the point of view of tax smoothing.

Although our analysis demonstrates that our generalization of Barro’s (1979) model provides a reasonable characterization of post-war U.S. policy, this need not be the case for other countries. In particular, some countries (e.g., Belgium, Canada, and Italy) experienced much larger increases to their debt–GDP levels during the 1980s than did the U.S., and these increases may well reflect the political constraints suggested by Alesina and Tabellini (1990) and Alesina and Perotti (1995). In a related paper, we assess the extent to which the fiscal policies of other OECD economies conform to our extended tax-smoothing model.

APPENDIX

Proof of Propositions

PROOF OF PROPOSITION 2: We need to show that the tax rate given by Equation (20) satisfies the first-order condition (Equation 8) and the resulting debt–GDP ratio satisfies the transversality condition. Leading Equation (20) forward one period and

taking condition expectations, noting that $E_t[q_{t+1}] = 1$, $E_t[q_{t+1}e^{r_t-\bar{v}+\gamma}] = e^{r_t-\bar{v}+\gamma}$, and that q_{t+1} , r_{t+1} , and g_{t+1} are independent, we have

$$\begin{aligned} E_t[q_{t+1}\tau_{t+1}] &= E_t[q_{t+1}\phi(r_{t+1})e^{r_t-\bar{v}+\gamma}]b_{t+1} + E_t[q_{t+1}]\bar{g} \\ &\quad + E_t[q_{t+1}\psi(r_{t+1})(g_{t+1} - \bar{g})] \\ &= e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]b_{t+1} + \bar{g} + E_t[\psi(r_{t+1})]\rho_g(g_t - \bar{g}). \end{aligned} \quad (34)$$

So, the first-order condition (Equation 8) is satisfied if and only if

$$\tau_t = e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]b_{t+1} + \bar{g} + E_t[\psi(r_{t+1})]\rho_g(g_t - \bar{g}). \quad (35)$$

From Equation (2), Equation (35) is equivalent to

$$\begin{aligned} \tau_t &= \frac{e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]}{1 + e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]} e^{r_{t-1}-\bar{v}_t} b_t + \bar{g} \\ &\quad + \frac{e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})] + E_t[\psi(r_{t+1})]\rho_g}{1 + e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]}. \end{aligned} \quad (36)$$

From Equations (19) and (22), however, we have

$$\frac{e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]}{1 + e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]} = \phi(r_t), \quad (37)$$

$$\begin{aligned} \frac{e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})] + E_t[\psi(r_{t+1})]\rho_g}{1 + e^{r_t-\bar{v}+\gamma} E_t[\phi(r_{t+1})]} &= (1 - \phi(r_t))\rho_g E_t[\psi(r_{t+1})] \\ &\quad + E_t[\psi(r_{t+1})] = \psi(r_t). \end{aligned} \quad (38)$$

Thus, Equation (36) is equivalent to Equation (20), which implies that the optimal tax rate satisfies the first-order condition (Equation 8). To verify that the transversality condition is satisfied, all we need to show is that the debt–GDP ratio grows at a rate that is strictly lower than the growth-adjusted interest rate. First, use Equation (20) to substitute for τ_t into Equation (2). This yields

$$b_{t+1} = (1 - \phi(r_t))e^{r_{t-1}-\bar{v}_t}b_t + (1 - \psi(r_t))(g_t - \bar{g}). \quad (39)$$

Given that $\phi(r_t) \geq \phi^* > 0$, we have $(1 - \phi(r_t))e^{r_{t-1}-\bar{v}_t} \leq (1 - \phi^*)e^{r_{t-1}-\bar{v}_t}$, so that the debt–GDP ratio implied by Equation (20) indeed grows at a rate that is strictly less than the growth-adjusted interest rate. Finally, observe that if there exists a strictly positive unique solution to Equation (19) then $0 \leq (1 - \phi(r_t))\rho_g < 1$. It follows that Equation (22) can be solved forward to get the unique function

$$\psi(r_t) = \phi(r_t) + \sum_{i=1}^{\infty} \rho_g^i E_t \left[\phi(r_{t+i}) \prod_{j=1}^i (1 - \phi(r_{t+j-1})) \right]. \quad (40)$$

Q.E.D.

PROPOSITION 3: Define the mapping T as follows:

$$(T\phi)(r) = \frac{\exp(r - \bar{v} + \gamma)E[\phi(r') \mid r]}{1 + \exp(r - \bar{v} + \gamma)E[\phi(r') \mid r]} . \tag{41}$$

If there exists a $\delta > 0$ such that $r - \bar{v} + \gamma \geq \delta$, then there exists a function ϕ that is a fixed point of T such that $1 > \phi \geq \phi^*$ for some $\phi^* > 0$.

PROOF: Let $\phi^* = 1 - \exp(-\delta) > 0$, and let D be the space of measurable functions of r such that $1 \geq \phi(r) \geq \phi^*$ for all r . Then D is a complete norm space with the sup-norm. For any $\phi \in D$, we have, from the condition in the proposition,

$$(T\phi)(r) \geq \frac{\exp(r - \bar{v} + \gamma)\phi^*}{1 + \exp(r - \bar{v} + \gamma)\phi^*} \geq \frac{\exp(\delta)\phi^*}{1 + \exp(\delta)\phi^*} = \phi^* . \tag{42}$$

So $T(D) \subseteq D$. It is clear that T is also a monotone operator and $T\phi^* \geq \phi^*$. From Theorem 17.7 of Stokey and Lucas (1989), $\phi = \lim_{n \rightarrow \infty} T^n\phi^*$ is a fixed point of T in D . Finally, from the fact that $\phi > 0$ and $T\phi = \phi$, we can see that $\phi < 1$. Q.E.D.

Numerical Algorithm for Solving ψ and ϕ

The algorithm we use to solve the function ϕ follows the proof of Proposition 3. Starting from $\phi^{(0)} = 1$, we let $\phi^{(n)} = T\phi^{(n-1)}$, and iterate until the sequence $\{\phi^{(n)}\}$ converges. Given ϕ , the function ψ is solved using the same algorithm, except that the operator T is now defined as follows:

$$(T\psi)(r) = (1 - \phi(r))\rho_g E[\psi(r') \mid r] + \phi(r) . \tag{43}$$

Analytical Examples

Example 1: If $r_t = \bar{r}, \forall t$, and inflation zero the solution to Equation (19) is given by

$$\phi(\bar{r}) = 1 - e^{-(\bar{r} - \bar{v} + \gamma)} , \tag{44}$$

and the solution to Equation (22) by

$$\psi(\bar{r}) = \frac{\phi(\bar{r})}{1 - (1 - \phi(\bar{r}))\rho_g} . \tag{45}$$

In the absence of spending shocks, the optimal growth in the debt-GDP ratio is then given by Equation (25).

Example 2: If $\rho_g = 0$, then from Equation (22), $\psi(r_t) = \phi(r_t)$, and hence Equation (26) follows.

Example 3: If $v_t = 0$ and $g_t = \bar{g}$ for all t , and $\sigma_m^2 = 0$, then, $\theta_t = \phi(r_t)e^{r_t - 1}$ and $\tau_t = \bar{g} + \phi(r_t)e^{r_t - 1}b_t$. Equation (19) becomes

$$\phi(r_t) = \frac{e^{r_t} E_t[\phi(r_{t+1})]}{1 + e^{r_t} E_t[\phi(r_{t+1})]} . \tag{46}$$

Assume further that the interest rate r_t follows a two-state Markov process with a state space $\{r_l, r_h\}$, where $r_l < r_h$. Let $\Pr[r_{t+1} = r_l | r_t = r_l] = p_l$ and $\Pr[r_{t+1} = r_h | r_t = r_h] = p_h$ be the transition probabilities, where $0 < p_l < 1$ and $0 < p_h \leq 1$. Then, ϕ can take on two values, $\phi_l = \phi(r_l)$ and $\phi_h = \phi(r_h)$, which are determined by the following equations:

$$\phi_l = \frac{e^{r_l}[p_l\phi_l + (1 - p_l)\phi_h]}{1 + e^{r_l}[p_l\phi_l + (1 - p_l)\phi_h]}, \quad (47)$$

and

$$\phi_h = \frac{e^{r_h}[p_h\phi_h + (1 - p_h)\phi_l]}{1 + e^{r_h}[p_h\phi_h + (1 - p_h)\phi_l]}. \quad (48)$$

As long as $r_l > 0$, there exist unique solutions to the above two equations and they satisfy the conditions in Proposition 2. In addition, let $\phi_s^* = 1 - e^{-r_s}$, $s \in \{l, h\}$. Then, it is straightforward to verify that the solutions to Equations (47) and (48) satisfy

$$\phi_l^* < \phi_l < \phi_h \leq \phi_h^*, \quad (49)$$

and that $\phi_h = \phi_h^*$ if and only if the transition to the high interest state is permanent, $p_h = 1$.

If $r_t = r_{t-1} = r_l$, then

$$\theta_t = \phi_l \exp(r_l) > \phi_l^* \exp(r_l) = \hat{r}_t \quad (50)$$

and therefore the debt-GDP ratio decreases. On the other hand, if $r_{t-1} = r_t = r_h$, we have

$$\theta_t = \phi_h \exp(r_h) \leq \phi_h^* \exp(r_h) = \hat{r}_t \quad (51)$$

and the equality holds if and only if $p_h = 1$.

Example 4: Everything is the same as in Example 3, except v_t is an i.i.d. variable and $p_h = 1$. Then, ϕ_h is given by

$$\phi_h = 1 - \exp(-(r_h - \bar{v})), \quad (52)$$

and ϕ_l is determined by the following equation:

$$\phi_l = \frac{\exp(r_l - \bar{v})[p_l\phi_l + (1 - p_l)\phi_h]}{1 + \exp(r_l - \bar{v})[p_l\phi_l + (1 - p_l)\phi_h]}. \quad (53)$$

If $r_{t-1} = r_t = r_h$, then the growth in the debt-GDP ratio is given by Equation (27).

The Data

All fiscal variables, including tax revenues, government expenditures, debt, and interest payments on debt, are taken from *The Economic Report of President, 2000*.

All these variables are based on fiscal years. To be consistent, the GDP data we use are also based on fiscal years and are taken from *The Economic Report of President, 2000* as well. Below are more detailed documentation of each of the variables we used in the paper. All the tables we refer to are those from *The Economic Report of President, 2000*, unless stated otherwise.

Tax revenues: *Total Receipts* from Table B-78, p. 399.

Government expenditures: *Total Outlays* minus *Net interest*, both from Table B-78, p. 399.

Public debt: *Federal Debt (end of period) Held by the public*, from Table B-76, p. 397.

GDP ($P_t Y_t$): *Gross domestic product* from Table B-76, p. 397.

Budget surplus: *Total Receipts* minus *Total Outlays*.

Primary surplus: Tax revenues minus government expenditures.

Tax rate (τ_t): Tax revenues divided by GDP.

Government expenditure–GDP ratio (g_t): Government expenditures divided by GDP.

Debt–GDP ratio (b_{t+1}): Public debt divided by GDP.

Effective interest rate on debt (r_t'): *Net interest* divided by last year's public debt. The risk-free interest rate is calculated as $r_t = \ln(1 + r_t')$.

Returns on market portfolio ($r_{e,t}$): Annual value weighted returns from CRSP.

Estimating σ_e^2 and σ_{ev}

Since GDP is measured as a flow during a fiscal year (from October 1 to September 30) and the market return is measured as the end of each calendar year, there is a mismatch in timing between the GDP growth rate and the stock return. We, therefore, use the one-year-lagged stock return rather than the current return in estimating the covariance between the GDP growth rates and the innovations in stock returns. This time convention is the same as that used in Campbell, Lo, and MacKinlay (1997, p. 308) in calculating the covariance between consumption growth and stock return. More specifically, we first run the following regression

$$r_{e,t-1} = \alpha_0 + \alpha_1 v_{t-1} + \alpha_2 r_{e,t-2} + \varepsilon_{e,t},$$

and then estimate the covariance σ_{ev} by the sample covariance between $\varepsilon_{e,t}$ and v_t . σ_e^2 is simply estimated by the sample variance of $\varepsilon_{e,t}$.

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