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Endogenous capital utilization, investor's effort, and optimal fiscal policy

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Abstract

The present paper studies the implications of endogenous capital utilization and investor's effort on the structure and time consistency of optimal fiscal policy. The main results are that levies of old capital are distortionary and should therefore not be used at confiscatory rates, that in a steady state capital and labor income should be taxed at the same rate and investment should be subsidized at the same rate as the tax rate, and that the optimal tax policy can be made time-consistent through debt restructuring.

Key words: Endogenous capital utilization; Investor's effort; Optimal fiscal policy; Time consistency

JEL classification: E22; E62; H21

1. Introduction

Traditional analyses of optimal capital income taxation have assumed that the utilization rates of capital are fixed and that the effectiveness of investment is independent of investor's effort.[1] Under these assumptions, levies on 'old'

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[1] There is an extensive literature on optimal capital income taxation in public finance which includes among others Atkinson (1971), Diamond (1973), Pestieau (1974), and Atkinson and Sandmo (1980). Most of these studies use overlapping generation models. Recent studies using infinite-horizon models include Judd (1989), Chamley (1986), Chari, Christiano, and Keheo (1994), King (1990), and Zhu (1992). For analyses of optimal taxation of both physical and human capital, see Lucas (1990), Jones, Manuelli, and Rossi (1990), and Yuen (1990).

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capital (capital that is accumulated previously) through capital income taxes or investment tax credits are nondistortionary since old capital is inelastically supplied. As a result, it has been shown that the optimal tax policy involves taxing income from inelastically supplied old capital at confiscatory rates but taxing income from future capital at low rates to encourage investment (see Chamley, 1986; Judd, 1985), and it has also been pointed out that the optimal tax policy is generally time-inconsistent because of its asymmetric treatment of income from old and new capital (see Kydland and Prescott, 1977; Fischer, 1980). Furthermore, it has been argued that the debt restructuring method suggested by Lucas and Stokey (1983) cannot solve the time inconsistency problem in economies with privately owned capital because of the capital levy issue.

The present paper will argue that both assumptions used in the traditional analyses are not valid and that implicit levies on old capital through either capital income taxes or investment tax credits *are* distortionary. This has important implications for both the structure and time consistency of optimal tax policy. In particular, it will be shown in the present paper that income from old capital should generally not be taxed at confiscatory rates, and that the optimal tax policy can be made time-consistent through debt restructuring.

Taxes on income from old capital have been considered to be lump-sum taxes because of the assumption that capital utilization rates are fixed. Under this assumption, taxes on income from previously accumulated capital, old capital, are taxes on pure economic rents and are thus nondistortionary. In the real world, however, capital utilization rates are not fixed.[2] For a given stock of capital, the utilization rate of capital is chosen such that the marginal return to the use of capital equals the marginal user cost. Capital income taxes, whether they are applied to old or new capital, introduce wedges between these two margins and are therefore distortionary. Investment tax credits, used along with capital income taxes, have also been considered as implicit capital levies that are nondistortionary: By reducing the cost of acquiring new capital, investment tax credits effectively reduce the market value of old capital and, with 100% tax credits, reduce the effective tax rates on returns to investment to zero. Not all of the costs of investment can be directly subsidized, however. Investment requires as input not only market goods but also investors' own effort in investment related activities. These activities include among others investor/entrepreneurs' searching for new investment opportunities and investor/shareholders' managing or monitoring their own investment projects.[3] The costs of investor's effort in these activities

[2] The importance of variations in the capital utilization rates in explaining aggregative economic activities has been recently advanced by Kydland and Prescott (1988, 1991), Greenwood, Hercowitz, and Huffman (1988), and Finn (1992).

[3] While there have been no studies (that the author is aware of) which directly measure the impact of investors' effort on the result of investment, it is clear that entrepreneurship is one of the most important factors that affect the effectiveness of capital accumulation, and the recent studies of

are generally hard if not impossible for the government to measure and therefore cannot be directly subsidized by the government.[4] In practice, then, a realistic tax system can only give tax credits to investors for their investment in market goods but not for their costs of undertaking effort in investment-related activities. As a result, investment tax credits cause investors to substitute market goods for their own effort in investment and are thus distortionary.

In the present paper, both capital utilization and investor's effort will be explicitly modeled to analyze their implications for the structure and time consistency of optimal capital income taxation. The model economy is presented in Section 2. As in most of the studies of optimal dynamic taxation, it is assumed that the amount of government consumption in each period is exogenously given and that the government finances it through flat-rate taxes on capital and labor income. The government is allowed to issue public debt to spread excess burden over time and to use investment tax credits to change the effective tax rates on returns to investment. It is shown in this section that capital income tax and investment tax credit are always distortionary and should therefore not be set at confiscatory levels. This finding is in stark contrast to the conclusion of previous studies that 'old' capital should be taxed at confiscatory rates and is a direct result of modeling capital utilization and investor's effort endogenously.

In Section 3, the structure of optimal capital and labor income taxation under full commitment is analyzed. The main finding of this section is that if the economy converges to a steady state, then, in the steady state capital and labor income should be taxed at the same rate and investment should be subsidized at the same rate as the tax rate. This finding implies that the optimal tax policy maintains aggregate production efficiency in the steady state, which can be viewed as an extension of Diamond and Mirrlees' (1971) result on production efficiency in static economies to the dynamic economy considered in the present paper. The finding also implies that the optimal effective tax rates on the returns to investment are zero, which is an extension of Chamley's (1986) result. The transitional dynamics of the optimal tax policy under full commitment is also characterized for certain class of utility functions.

The issue of time inconsistency is taken up in Section 4. In a seminal paper, Lucas and Stokey (1983) show that, in an economy *without capital*, the optimal tax policy can be made time-consistent if the government commits to honoring public debt and if a debt of sufficiently rich maturity structure can be

management buyouts (MBO) by Kaplan (1989) and Smith (1990) and of venture capital investment by Barry et al. (1990) and Sahlman (1990) provide indirect evidence on the importance of managerial and monitoring efforts by investors in improving the effectiveness of investment.

[4] A unique feature of investors' effort in investment-related activity that distinguishes it from standard labor effort is that it is generally rewarded by the increase in the value of investment which depends both on the input of market goods and effort rather than by wage payments which depend on effort alone.

issued. Their result is later extended by Persson, Persson, and Svensson (1987) to a monetary economy and by Faig (1990) to the case when government consumption is endogenously determined and when capital is exclusively owned by the government. But for economies with privately owned capital, it has been concluded by these authors that the debt restructuring method cannot make the optimal tax policy time-consistent because of the capital levy problem. However, when endogenous capital utilization and investor's effort are explicitly taken into account implicit capital levies through capital income taxes or investment tax credits are distortionary and, as a result, debt restructuring can be used to make the optimal tax policy time-consistent even in economies with capital. In Section 4 this is shown for the model economy that is considered in the present paper, with the assumption that the government can issue debt indexed to any of the following variables: (i) aggregate consumption, (ii) average after-tax wage rate, and (iii) average after-tax return to capital.[5] The analysis in this section shows that the problem of time inconsistency in economies with capital is *not* fundamentally different from that in economies without capital, and it is complementary to the previous analyses of time inconsistency problem in economies without capital.

Conclusions are given in Section 5.

2. The model economy and competitive equilibrium

2.1. The model economy

Consider an economy that is populated with a large number of identical, infinitely lived households who derive utility from consumption and leisure. A typical household's preference over lifetime consumption and leisure is represented by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad 0 < \beta < 1, \tag{1}$$

where c_t and l_t are the household's consumption and leisure (measured as a fraction of its time endowment, which is normalized to 1) in period t, $u(.,.)$ is the period utility function, and β is the discount factor. In this economy there is only one final good which can either be consumed or invested. The final good can be produced using capital and labor services with a constant return to scale production technology,

$$y_t = F(s_t, n_t),$$

[5] See the end of Section 4 for a discussion of the feasibility of issuing debt with such structures.

where s_t and n_t are the capital and labor services employed in final good production in period t. It is assumed that $s_t = h_t k_t$, where k_t is the capital stock and h_t is the capital utilization rate in period t.

Following Taubman and Wilkinson (1970) and Greenwood, Hercowitz, and Huffman (1988), the present paper models the rate of capital utilization as the work-hours of capital and assumes that the only cost associated with capital utilization is a depreciation cost. Let $\delta(h)$ be the depreciation function that determines the rate of depreciation of capital for a given capital utilization rate, h. It is assumed that $\delta(.)$ is a strictly increasing and strictly convex function. This assumption captures the notion that longer work-hours of capital causing faster depreciation of the capital stock at an increasing rate because of wear and tear.

Capital can be produced using the final good and *investor's effort*.[6] So, the evolution of capital accumulation is described by the following equation:

$$k_{t+1} = (1 - \delta(h_t))k_t + I(x_t, e_t), \tag{2}$$

where $I(.,.)$ is a constant return to scale capital production function, x_t the investment in the final good, and e_t the investor's effort in period t.

To ensure interior solutions, we assume throughout the paper that both $F(.,.)$ and $I(.,.)$ are strictly increasing and concave functions that satisfy the following Inada type conditions:

$$\lim_{s \to 0} F_1(s,n) = \lim_{n \to 0} F_2(s,n) = \lim_{x \to 0} I_1(x,e) = \lim_{e \to 0} I_2(x,e) = +\infty.$$

In each period, the government consumes a fixed amount of the final good. Let g_t denote the government's consumption in period t; then, the economy's resource constraints are

$$c_t + g_t + x_t \le F(h_t k_t, n_t), \tag{3}$$

$$l_t + n_t + e_t = 1. \tag{4}$$

In the present paper, it is assumed that to finance its consumption the government cannot use lump-sum taxes but only flat-rate taxes, and that only taxes on quantities that involve *market transactions* can be used. In this economy, the quantities that involve market transactions are consumption, capital and labor services, and investment. It will be assumed that flat-rate taxes on income from capital and labor services and tax credits on investment expenditures can be used by the government but taxes on consumption cannot.[7]

[6] One could have assumed that the production of capital requires capital and labor services as input as well. That would not alter the characteristics of the results in the present paper but make expositions much more complicated and less transparent.

[7] In this representative agent model economy, the first-best allocation can be achieved if the government can use all four tax instruments: flat-rate taxes on consumption and income from capital

The government is allowed to issue debt with a sufficiently rich maturity and payoff structure. In particular, it is assumed that the government can issue debt that has its payoff indexed to consumption, to *average after-tax wage rate*, and to *average after-tax return to capital*, and that debt indexed to consumption and average after-tax wage rate can be issued with any maturities. When the full commitment technology is available, the maturity and payoff structure of public debt is irrelevant, and the optimal tax policy can be implemented with one-period, consumption-denominated debt alone. But it will be shown in Section 4 that, when full commitment technology is not available, the government's ability to issue debt with sufficiently rich maturity and payoff structure is of vital importance in implementing the optimal tax policy.

At time 0, the debt structure that the current government inherits from the previous government is represented by b_0^R and the sequence $\{b_{0t}^c, b_{0t}^w\}_{t\geq 0}$. Here, b_0^R represents the amount of debt that has to be paid in period 0 denominated in average after-tax return to capital, and b_{0t}^c and b_{0t}^w the amount of debt that has to be paid in period t denominated in consumption and after-tax wage rate respectively.

2.2. *Competitive equilibrium*

The economy is decentralized with perfectly competitive markets: labor market and rental market where firms hire labor and rent capital services respectively from households for production of the final good, capital good market where capital can be exchanged for the final good, spot and forward markets for the final good, and government bond market where bonds with different maturities and denominations are traded. Let w_t be the wage rate, r_t the rental rate of capital services, q_t the price of capital, all denominated in the final good in period t. Let τ_t be the tax rate on capital income, θ_t the tax rate on labor income, and π_t the investment tax credit. And let p_t be the time 0 forward price of the final good delivered in period t. For now, it is assumed that the current government has the technology to make future governments fully committed to current policy. Then, taking prices and the government's fiscal policy as given, a typical household's problem is choosing consumption c_t, labor supply for the final good production n_t, investment x_t, investment-related effort e_t, capital utilization rate h_t, and capital stock k_{t+1} for every $t \geq 0$ to maximize (1) subject to the

and labor, and tax credits on investment expenditures. The government can do so simply by setting all the taxes and credits at the same, constant rate. To focus on the interesting cases when the government has to use distortionary taxes, we assume that the government can only use three of the four tax instruments. In can be easily shown that all combinations of three tax instruments are equivalent in that they have the same set of implementable allocations. To be comparable with the literature on dynamic taxation, we choose consumption tax as the one that cannot be used by the government.

budget constraint

$$\sum_{t=0}^{\infty} p_t\{c_t - b_{0t}^c + (1-\pi_t)x_t + (1-\pi_t)q_t[k_{t+1} - (1-\delta(h_t))k_t - I(x_t, e_t)]\}$$

$$\leq p_0 R_0 b_0^R + \sum_{t=0}^{\infty} p_t\{(1-\tau_t)r_t h_t k_t + (1-\theta_t)w_t(n_t + b_{0t}^w)\}. \tag{5}$$

Here, R_0 is the *average* after-tax return to capital in period 0.

Note that by writing the above constraint for the household it has been implicitly assumed that the production of capital is done within the household rather than in an outside firm. This is equivalent to assuming that the capital is produced in a firm but the household is the owner or shareholder of the firm. What is emphasized here is that the household/investor's effort is an important input for the capital production and the reward to this input is through the increase in the value of capital. (See Footnotes 3 and 4 in Section 1 above.)

A typical final good producing firm's problem in this economy is maximizing profits taking factor prices as given. This leads to the following first-order conditions:

$$r_t = F_1(s_t, n_t), \quad w_t = F_2(s_t, n_t). \tag{6}$$

The competitive equilibrium in this economy is defined as follows. For given b_0^R, $\{g_t, b_{0t}^c, b_{0t}^w, \pi_t, \tau_t, \theta_t\}_{t\geq 0}$, an allocation $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 0}$ is a competitive equilibrium allocation if and only if there exists a price sequence $\{p_t, r_t, w_t, q_t\}_{t\geq 0}$ such that the allocation solves the household's maximization problem and satisfies the equilibrium condition (6) and the resource constraints (2) through (4).

To characterize competitive allocations, then, one needs to characterize the solutions to the household's maximization problem. Assuming that an interior solution to the household's maximization problem exists, then, the solution is characterized by the budget constraint (5) with equality, the transversality condition

$$\lim_{t\to\infty} p_t(1-\pi_t)q_t k_{t-1} = 0, \tag{7}$$

and the following first-order conditions: for every $t \geq 0$,

$$\frac{p_t}{p_0} = \beta^t \frac{u_1(t)}{u_1(0)}, \tag{8}$$

$$q_t = I_1^{-1}(t), \tag{9}$$

$$(1-\theta_t)w_t = (1-\pi_t)\frac{I_2(t)}{I_1(t)} = \frac{u_2(t)}{u_1(t)}, \tag{10}$$

$$(1-\tau_t)r_t = (1-\pi_t)q_t\delta'(h_t), \tag{11}$$

$$u_1(t)(1-\pi_t)q_t$$
$$= \beta u_1(t+1)[(1-\pi_{t+1})q_{t+1}(1-\delta(h_{t+1}))+(1-\tau_{t+1})r_{t+1}h_{t+1}]. \qquad (12)$$

Here, and for the rest of the paper, $u_1(t)$ denotes $u_1(c_t, l_t)$, $u_2(t)$ denotes $u_2(c_t, l_t)$, $I_1(t)$ denotes $I_1(x_t, e_t)$, etc.

These first-order conditions show immediately that there are no lump-sum taxes in this economy. Investment tax credits distort the household's choice between the amount of final good committed to investment and the level of efforts in investment-related activities. And capital income taxes distort the choice of capital utilization rates. Since it has been assumed that $I(.,.)$ is strictly concave and homogeneous of degree one, the ratio $I_2(t)/I_1(t)$ is an increasing function of the investment/effort ratio, x_t/e_t, alone. Eq. (10) then shows that the household's investment/effort ratio is a function of both after-tax wage rate and investment tax credit. An increase in the labor income tax rate reduces the after-tax wage rate and induces investors to work harder on their investment rather than work for wage income. On the other hand, an increase in the investment tax credit decreases the cost of using market goods in investment relative to the cost of undertaking investment-related effort and induces investors to work less hard on their investment. Thus, the distortion of investment/effort ratio depends on the relative magnitude of the labor income tax rate and investment tax credit. Similarly, Eq. (11) shows that the distortion of capital utilization depends on the relative magnitude of the capital income tax rate and investment tax credit. These distortions exist not only in period $t \geq 1$ but also in period 0. Therefore, it is generally not optimal to set the capital income tax rate or investment tax credit rate at a confiscatory level in the initial period since they are distortionary. This is in stark contrast with the conclusion reached by the previous studies that assume fixed capital utilization rates and that the effectiveness of investment is independent of investor's effort.[8]

The next section turns to the problem of characterizing the optimal fiscal policy with full commitment.

3. Optimal fiscal policy with full commitment

In a competitive equilibrium, the household who chooses the capital utilization rate h_0 in the initial period has the after-tax return to capital $R_0 = (1-\tau_0)r_0h_0+$

[8] If capital utilization rates are fixed, then the first-order condition (11) would not be present and the household's maximization behavior does not impose any constraints on the level of the capital income tax rate in the initial period. Similarly, if the effectiveness of investment is independent of investor's effort, i.e., $I(.,.)$ is a function of x_t only, then the second part of the first-order condition (10) would not be present and the household's maximization behavior does not impose any constraints on the level of the investment tax credit in the initial period. In either case, capital income tax rate in the initial period can be set at a confiscatory level without causing any distortions.

$(1-\pi_0)(1-\delta(h_0))q_0$. Since the economy consists of identical households, the average level of capital utilization rate is h_0 and average after-tax return to capital in period 0 is also R_0. Substituting R_0 and (2) into (5) and using (6) to eliminate price and tax variables from the equilibrium conditions (5) and (7) through (12), one gets

$$\lim_{t\to\infty} \beta^t \frac{u_2(t)}{l_2(t)} k_{t+1} = 0, \tag{13}$$

$$\frac{u_2(t)}{l_2(t)} = \beta \frac{u_2(t+1)}{l_2(t+1)} [1 - \delta(h_{t+1}) + \delta'(h_{t+1})h_{t+1}], \quad \forall t \geq 0, \tag{14}$$

$$\sum_{t=0}^{\infty} \beta^t [u_1(t)(c_t - b_{0t}^c) - u_2(t)(n_t + e_t + b_{0t}^w)]$$
$$= \frac{u_2(0)}{l_2(0)} [1 - \delta(h_0) + \delta'(h_0)h_0](k_0 + b_0^R). \tag{15}$$

So, for given b_0^R, $\{g_t, b_{0t}^c, b_{0t}^w\}_{t\geq 0}$, an allocation $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 0}$ can be implemented by the government through some tax policy $\{\pi_t, \tau_t, \theta_t\}_{t\geq 0}$ only if it satisfies the resource constraints (2) through (4) and conditions (13) through (15). On the other hand, for any allocation that satisfies (2) through (4) and (13) through (15), one can easily show that the government can choose a tax policy to implement the allocation in a competitive equilibrium. Such an allocation is called an *implementable allocation.* A benevolent government's objective is to choose an optimal implementable allocation that maximizes (1).

Using an argument similar to the one that is used in Zhu (1992), one can show that the first-order conditions for the government's optimization problem can be derived by using a generalized Kuhn–Tucker theorem. For every $t \geq 0$, let μ_t, λ_t, and ϕ_t be the Lagrange multiplier for constraints (2), (3), and (14) respectively. Let Φ be the Lagrange multiplier for constraint (15) and let

$$\phi_{-1} = \beta^{-1}\Phi(k_0 + b_0^R).$$

Then, the first-order conditions for the government's optimization problem are, for every $t \geq 0$,

$$\lambda_t = \beta^t(1+\Phi)u_1(t) + \beta^t\Phi[u_{11}(t)(c_t - b_{0t}^c) - u_{12}(t)(n_t + e_t + b_{0t}^w)]$$
$$+ \frac{u_{12}(t)}{l_2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{16}$$

$$\lambda_t F_2(t) = \beta^t(1+\Phi)u_2(t) + \beta^t\Phi[u_{12}(t)(c_t - b_{0t}^c) - u_{22}(t)(n_t + e_t + b_{0t}^w)]$$
$$+ \frac{u_{22}(t)}{l_2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{17}$$

$$\mu_t I_2(t) = \beta^t (1 + \Phi) u_2(t) + \beta^t \Phi [u_{12}(t)(c_t - b_{0t}^c) - u_{22}(t)(n_t + e_t + b_{0t}^w)]$$
$$+ \frac{u_{22}(t)}{I_2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)]$$
$$+ \frac{u_2(t) I_{22}(t)}{I_2^2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{18}$$

$$\mu_t I_1(t) - \lambda_t = \frac{u_2(t) I_{12}(t)}{I_2^2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{19}$$

$$[\lambda_t F_1(t) - \mu_t \delta'(h_t)]k_t = \beta\phi_{t-1}\frac{u_2(t)}{I_2(t)}\delta''(h_t)h_t, \tag{20}$$

$$\mu_t = \mu_{t+1}(1 - \delta(h_{t+1})) + \lambda_{t+1} F_1(t+1) h_{t+1}, \tag{21}$$

$$\lambda_t [F(t) - c_t - x_t - g_t] = 0. \tag{22}$$

These first-order conditions, the resource constraints (2) through (4), and the equilibrium conditions (13) through (15) are necessary conditions for the optimal implementable allocation (if one exists). For the rest of the paper, any allocation that satisfies these necessary conditions is called a *Ramsey allocation*, the corresponding tax policy that implements the allocation the *Ramsey policy*, and the corresponding competitive equilibrium the *Ramsey equilibrium*.

What can one say about the Ramsey policy? The following proposition shows that when the Ramsey allocation converges to a steady state, the optimal capital and labor income tax rates converge to the same value, and that the optimal level of investment tax credit converges to the same value as the optimal tax rates do.

Proposition 1. If a Ramsey allocation converges to a steady state, then:
(i) $\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \tau_t = \lim_{t\to\infty} \theta_t$.
(ii) *The optimal effective tax rate on the return to investment is zero in the steady state.*
(iii) *The aggregate production efficiency is maintained in the steady state.*

Proof. A proof of (i) is given in the Appendix. Since the before- and after-tax returns to investment in period t are

$$R_t^I = \frac{r_{t+1}h_{t+1} + [1 - \delta(h_{t+1})]/I_1(t+1)}{I_1^{-1}(t)},$$

$$\bar{R}_t^I = \frac{(1 - \tau_{t+1}) r_{t+1} h_{t+1} + (1 - \pi_{t+1})[1 - \delta(h_{t+1})]/I_1(t+1)}{(1 - \pi_t) I_1^{-1}(t)},$$

(i) implies that $\lim_{t\to\infty} R_t = \lim_{t\to\infty} \bar{R}_t$ and thus (ii). Substituting (i) and (6) into the first-order conditions (10) and (11), one has that, in the steady state,

$$F_2(t) = \frac{l_2(t)}{l_1(t)}, \quad F_1(t) = \frac{\delta'(h_t)}{l_1(t)}.$$

Substituting (i) and the second equation above into the first-order condition (12), then, one has that, in the steady state,

$$u_1(t) = \beta u_1(t+1)\frac{F_1(t+1)h_{t+1} + [1-\delta(h_{t+1})]/I_1^{-1}(t+1)}{I_1^{-1}(t)} = \beta u_1(t+1)R_t^I.$$

Hence, the aggregate production efficiency is maintained in the steady state and thus (iii) holds. Q.E.D.

The result (ii) of Proposition 1 is, in some sense, an extension of Chamley's (1986) result. In the economy that Chamley considers, there are no investment tax credits (they are redundant instruments when capital utilization rates are fixed and the effectiveness of investment is independent of investor's effort). Thus, the effective tax rate on the return to investment is the same as the capital income tax rate in that economy, and he shows that in the steady state the optimal capital income tax rate is zero, which implies that the before- and after-tax returns to investment are the same in the steady state.

Diamond and Mirrlees (1971) show that in static economies Ramsey allocation maintains aggregate production efficiency. The result (iii) of Proposition 1 is an extension of their result to the dynamic economy considered in the present paper.

The next proposition characterizes the transitional dynamics of the Ramsey policy for certain class of utility functions.

Proposition 2. If the Ramsey allocation converges to a steady state and $\bar{\theta}$ is the steady state value of the optimal tax rates, and if $b_{0t}^c = 0$, $t \geq 0$, and the period utility function is of the following form:

$$u(c_t, l_t) = \frac{1}{1-\sigma}c_t^{1-\sigma} + \alpha l_t, \quad \sigma > 0, \quad \sigma \neq 1, \quad \alpha > 0, \tag{23}$$

or

$$u(c_t, l_t) = \log(c_t) + \alpha l_t, \quad \alpha > 0, \tag{24}$$

then:

(i) *The optimal labor income tax rate θ_t reaches the steady state value $\bar{\theta}$ immediately.*

(ii) *The capital income tax rate τ_t and the investment tax credit π_t reach the steady state value in finite periods if and only if $b_0^R + k_0 = 0$, in which case they reach the steady state value immediately.*

(iii) *If* $b_0^R + k_0 \geq 0$ *(or* < 0*), then both* τ_t *and* π_t *will converge to the steady state value* $\bar{\theta}$ *from above (or below).*

Proof. See Appendix. Q.E.D.

For the same class of utility functions described by (23) and (24), Chamley's analysis implies that the optimal capital income tax rate reaches the steady state value in finite periods. Proposition 2 shows that, with endogenous capital utilization and investor's effort, the dynamics of the optimal capital income tax rate and investment tax credit is more complicated and only in some very special cases can they reach the steady state value in finite periods. Depending on the sign of $b_0^R + k_0$, the optimal capital income tax rate and the optimal level of investment tax credit will converge to the steady state value from above or below. The optimal labor income tax rate, however, reaches the steady state value in the first period.

4. Time consistency of optimal fiscal policy

In Section 3 it is shown that both the capital income tax and investment tax credit in the initial period are distortionary and should therefore not be used at confiscatory levels. The present section studies its implication for the time consistency of optimal fiscal policy.

Previous studies have concluded that in economies with privately owned capital the optimal tax policy cannot be made time-consistent because of the issue of capital levy. This conclusion is based on the assumption that implicit levies like capital income taxes and investment tax credits are nondistortionary. The present paper has just shown, however, that these implicit levies are distortionary. Therefore, the assumption that leads to the conclusion by previous studies is false and the time consistency of optimal tax policy in economies with capital needs to be reconsidered.

The fact that capital levies are distortionary alone, however, does not guarantee that the optimal tax policy will be time-consistent. As pointed out by Lucas and Stokey (1983), future governments may still want to deviate from the optimal tax policy because they may, through changing the tax policy, affect the real interest rates and therefore 'devalue' the outstanding public debt. On the other hand, the analyses by Lucas and Stokey (1983), Persson, Persson, and Svensson (1987), and Faig (1990) show that in economies where there is no privately owned capital, the optimal tax policy can be made time-consistent through debt restructuring. This is because that, when the public debt is carefully structured, its value can be neutral to changes in taxes and thus eliminate the devaluation incentives that cause the time inconsistency problem. In this section it will be shown that, if governments commit to honoring public debt and if debt denominated in average after-tax wage rate and average after-tax return to capital can be issued along with the debt denominated in consumption, then the optimal tax policy can be made time-consistent even in the economy with capital that is considered in the present paper. So, the result in this section is complementary to the previous analyses of the time inconsistency of optimal policy in economies with no privately owned capital.

Following Lucas and Stokey (1983), it will be assumed in this section that future governments do not commit themselves to the optimal tax policy chosen by the current government, but they do commit themselves to honoring the public debt they inherited from the previous government and they are free to restructure the debt. Under this assumption, it will be shown below that the current government can always restructure the outstanding public debt, leaving with the government at $t = 1$ a sufficiently rich structure of public debt such that the government at $t = 1$ will actually *choose* to continue the optimal tax policy that is chosen by the current government. In other words, the optimal tax policy *can* be made time-consistent through debt restructuring.

Formally, in order to prove this result one needs to show that if $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 0}$ is a Ramsey allocation at time 0 for the given government consumption stream $\{g_t\}_{t\geq 0}$ and the public debt outstanding b_0^R and $\{b_{0t}^c, b_{0t}^w\}_{t\geq 0}$, then, $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 1}$ is a Ramsey allocation at time 1 for the given $\{g_t\}_{t\geq 1}$ and some restructured debt b_1^R and $\{b_{1t}^c, b_{1t}^w\}_{t\geq 1}$.

Let $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 0}$ be a Ramsey allocation at time 0, i.e., it satisfies (2) through (4) and (13) through (15) for some Φ and $\{\mu_t, \lambda_t, \phi_t\}_{t\geq 0}$. If $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 1}$ is a Ramsey allocation at time 1 for some restructured debt b_1^R and $\{b_{1t}^c, b_{1t}^w\}_{t\geq 1}$, then, it must satisfy the following conditions for some Φ', $\{\mu_t', \lambda_t'\}_{t\geq 1}$, and $\{\phi_t'\}_{t\geq 0}$:

$$\phi_0' = \beta^{-1}\Phi'(k_1 + b_1^R), \tag{25}$$

and for every $t \geq 1$,

$$\lambda_t' = \beta^t(1 + \Phi')u_1(t) + \beta^t\Phi'[u_{11}(t)(c_t - b_{1t}^c) - u_{12}(t)(n_t + e_t + b_{1t}^w)] + \frac{u_{12}(t)}{l_2(t)}[\phi_t' - \beta\phi_{t-1}'(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{26}$$

$$\lambda_t' F_2(t) = \beta^t(1 + \Phi')u_2(t) + \beta^t\Phi'[u_{12}(t)(c_t - b_{1t}^c) - u_{22}(t)(n_t + e_t + b_{1t}^w)] + \frac{u_{22}(t)}{l_2(t)}[\phi_t' - \beta\phi_{t-1}'(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{27}$$

$$\begin{aligned}\mu_t' l_2(t) = {} & \beta^t(1 + \Phi')u_2(t) + \beta^t\Phi'[u_{12}(t)(c_t - b_{1t}^c) - u_{22}(t)(n_t + e_t + b_{1t}^w)] \\ & + \frac{u_{22}(t)}{l_2(t)}[\phi_t' - \beta\phi_{t-1}'(1 - \delta(h_t) + \delta'(h_t)h_t)] \\ & + \frac{u_2(t)l_{22}(t)}{l_2^2(t)}[\phi_t' - \beta\phi_{t-1}'(1 - \delta(h_t) + \delta'(h_t)h_t)],\end{aligned} \tag{28}$$

$$\mu'_t I_1(t) - \lambda'_t = \frac{u_2(t)I_{12}(t)}{I_2^2(t)}[\phi'_t - \beta\phi'_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{29}$$

$$[\lambda'_t F_1(t) - \mu'_t\delta'(h_t)]k_t = \beta\phi'_{t-1}\frac{u_2(t)}{I_2(t)}\delta''(h_t)h_t, \tag{30}$$

$$\mu'_t = \mu'_{t+1}(1 - \delta(h_{t+1})) + \lambda'_{t+1}F_1(t+1)h_{t+1}, \tag{31}$$

$$\lambda'_t[F(t) - c_t - x_t - g_t] = 0, \tag{32}$$

$$\sum_{t=1}^{\infty}\beta^{t-1}[u_1(t)(c_t - b^c_{1t}) - u_2(t)(n_t + e_t + b^w_{1t})]$$
$$= \frac{u_2(1)}{I_2(1)}[1 - \delta(h_1) + \delta'(h_1)h_1](k_1 + b^R_1). \tag{33}$$

But in the Appendix it is shown that this is true. Thus:

Proposition 3. If $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t\geq 0}$ *is a Ramsey allocation at* $t = 0$ *for the given government consumption stream* $\{g_t\}_{t\geq 0}$ *and public debt outstanding* b^R_0 *and* $\{b^c_{0t}, b^w_{0t}\}_{t\geq 0}$, *and if the utility function* $u(.,.)$ *is strictly concave, then there always exist some restructured debt* b^R_1 *and* $\{b^c_{1t}, b^w_{1t}\}_{t\geq 1}$, *and* Φ', $\{\mu'_t, \lambda'_t\}_{t\geq 1}$, *and* $\{\phi'_t\}_{t\geq 0}$ *such that (25) through (33) are satisfied by* $\{c_t, n_t, e_t, h_t, x_t, k_{t-1}\}_{t\geq 1}$.

Proof. See Appendix.

With Proposition 3, then, it has been shown that the optimal tax policy can be made time-consistent through debt restructuring.

At this point, it is important to point out that Proposition 3 is shown under the assumption that debt denominated in average after-tax wage rate and in average after-tax return to capital can be issued by the government. This raises the question about how practical it is for the government to issue debt with such a complex payoff structure. In recent years, however, derivative financial products have been increasingly used by participants in the world financial markets. Synthetic debt instruments which have payoffs indexed to exchange rates, to stock indices, to commodity prices, etc., have already been issued by many private financial institutions.[9] Given the increasing liquidity of the market for these financial products, it would not be impossible nor too costly for governments to issue debt with the structure that is described above. Furthermore, as has been pointed out by Faig (1990), in practice there exist government liabilities, such as the promise to pay future social security pensions, that are closely linked to average after-tax wage rate.

[9] See, e.g., Darby (1994) and Litzenberger (1992).

A more fundamental problem, however, is how can a government credibly commit to not defaulting its debt. But this problem exists for economies without capital as well. What the present paper shows is that the time-inconsistency problems in economies with capital is not fundamentally different from that in economies without capital.

5. Conclusions

The present paper studies the implications of endogenous capital utilization and investor's effort for the structure and time consistency of optimal fiscal policy. It is shown that levies of old capital are distortionary and should therefore not be used at confiscatory rates, and that the optimal tax policy *can* be made *time-consistent* through debt restructuring. It is also shown that in a steady state capital and labor income should be taxed at the same rate and investment should be subsidized at the same rate as the tax rate.

There are other potentially interesting implications of the model that are not explored here. One implication of the model is regarding the determinacy of optimal fiscal policy in stochastic economies. Recent studies of optimal fiscal policy in stochastic economies that implicitly assume fixed capital utilization rates and investor's effort levels have shown that the optimal *ex post* capital income tax rates are not determinant because capital income taxes only cause intertemporal distortions (see Chari, Christiano, and Kehoe, 1994; King, 1990; Zhu, 1992). When capital utilization is endogenously determined, however, capital income taxes cause not only intertemporal distortions but also *atemporal* distortions – the distortions of capital utilization and investors' choice of their effort level in investment related activities. Therefore, a stochastic version of the model presented in this paper would imply that the optimal fiscal policy is determinant.

Another implication of the model is regarding the welfare gains from a Ramsey tax reform. Using a standard real business cycle model, Chari, Christiano, and Kehoe (1994) show that about 80% of the welfare gains from a Ramsey tax reform would come from the high tax rate on incomes from old capital. In other words, heavy levy on old capital is, quantitatively, the most important part of the Ramsey tax reform. In the model presented in this paper, however, levies on old capital are distortionary, and therefore the welfare gains from a Ramsey tax reform may have to be revised downward significantly.

Appendix

This appendix contains proofs of propositions. The following lemmas will be useful for the proofs.

Lemma 1. If $I(.,.)$ is a strictly concave function of homogeneous degree one, then $I_1 I_{22} - I_2 I_{12} < 0$ and $I_{22} - I_{12} F_2 < 0$.

Proof. Since $I(.,.)$ is of homogeneous degree one, we have

$$I(x, e) = I_1 x + I_2 e.$$

Taking partial derivatives with respect to e on both side of the above equation yields

$$I_2 = I_{12} x + I_{22} e + I_2,$$

which implies

$$I_{12} = -I_{22} e/x.$$

Therefore, from the concavity of $I(.,.)$, we have

$$I_1 I_{22} - I_2 I_{12} = I_{22}[I_1 x + I_2 e]/x = I_{22} I/x < 0,$$

$$I_{22} - I_{12} F_2 = I_{22}(1 + F_2 e/x) < 0. \quad \text{Q.E.D.}$$

Lemma 2. If $\{c_t, n_t, e_t, h_t, x_t, k_{t+1}\}_{t \geq 0}$ is a Ramsey allocation that satisfies (2) through (4) and (13) through (15) for some Φ and $\{\mu_t, \lambda_t, \phi_t\}_{t \geq 0}$, then, for very $t \geq 0$,

$$\mu_t I_2(t) - \lambda_t F_2(t) = \frac{u_2(t) I_{22}(t)}{I_2^2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)], \tag{A.1}$$

and there is an $A_t > 0$, which is a function of $(n_t, e_t, h_t, x_t, k_t)$ only, such that $\mu_t = A_t \lambda_t$.

Proof. Eq. (A.1) results directly from substracting (17) from (18). Comparing (A.1) to (19), we have

$$[I_1(t) I_{22}(t) - I_2(t) I_{12}(t)]\mu_t = [I_{22}(t) - I_{12}(t) F_2(t)]\lambda_t$$

or

$$\mu_t = [I_1(t) I_{22}(t) - I_2(t) I_{12}(t)]^{-1}[I_{22}(t) - I_{12}(t) F_2(t)]\lambda_t.$$

Letting

$$A_t = [I_1(t) I_{22}(t) - I_2(t) I_{12}(t)]^{-1}[I_{22}(t) - I_{12}(t) F_2(t)] > 0,$$

then, $A_t > 0$ since, from Lemma 1, $I_1(t) I_{22}(t) - I_2(t) I_{12}(t) < 0$ and $I_{22}(t) - I_{12}(t) F_2(t) < 0$. Q.E.D.

Proof of Proposition 1. From Lemma 2, for every $t \geq 0$, there is an $A_t > 0$ such that $\mu_t = A_t \lambda_t$ and A_t is a function of $(n_t, e_t, h_t, x_t, k_t)$. Substituting it into (21), one has that

$$\lambda_t = [1 - \delta(h_{t+1}) + A_{t+1}^{-1} F_1(t+1) h_{t+1}] A_{t+1} A_t^{-1} \lambda_{t+1}. \tag{A.2}$$

When the economy converges to a steady state $\beta^{-t}\lambda_t$ converges to a positive constant $\bar{\lambda}$. Thus, multiplying the both sides of (A.2) by β^{-t} and taking limits yields

$$1 = \beta \lim_{t\to\infty} [1 - \delta(h_t) + A_t^{-1} F_1(t) h_t]. \tag{A.3}$$

But Eq. (14) implies

$$1 = \beta \lim_{t\to\infty} [1 - \delta(h_t) + \delta'(h_t) h_t]. \tag{A.4}$$

Hence, comparing (A.3) and (A.4), we have

$$\lim_{t\to\infty} A_t^{-1} F_1(t) = \lim_{t\to\infty} \delta'(h_t). \tag{A.5}$$

Substituting $\mu_t = A_t \lambda_t$ into (20), multiplying both sides of it by $\beta^{-t} A_t^{-1}$, and taking limits yields

$$\bar{\lambda} \lim_{t\to\infty} [A_t^{-1} F_1(t) - \delta'(h_t)] = \lim_{t\to\infty} \beta^{-t} \phi_t A_t^{-1} \frac{u_1(t)}{I_1(t)} \delta''(h_t) h_t,$$

which implies, from (A.5),

$$\lim_{t\to\infty} \beta^{-t} \phi_t = 0.$$

Now, multiplying both (19) and (A.1) by β^{-t} and taking limits yields

$$\bar{\lambda} \lim_{t\to\infty} [A_t I_1(t) - 1] = \lim_{t\to\infty} \beta^{-t} \phi_t \frac{u_2(t) I_{12}(t)}{I_{22}^2(t)} [\delta'(h_t) h_t - \delta(h_t)] = 0, \tag{A.6}$$

$$\bar{\lambda} \lim_{t\to\infty} [\mu_t I_2(t) - \lambda_t F_2(t)] = \lim_{t\to\infty} \beta^{-t} \phi_t \frac{u_2(t) I_{22}(t)}{I_{22}^2(t)} [\delta'(h_t) h_t - \delta(h_t)] = 0. \tag{A.7}$$

Substituting $\mu_t = A_t \lambda_t$ into (A.7) and comparing it to (A.6), we have

$$\lim_{t\to\infty} \frac{I_2(t)}{I_1(t)} = \lim_{t\to\infty} F_2(t).$$

Comparing (A.6) to (A.5), we have

$$\lim_{t\to\infty} I_1(t) F_1(t) = \lim_{t\to\infty} \delta'(h_t).$$

Finally, substituting the last two equations and (6) into (10) and (11) respectively, we have

$$\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \tau_t = \lim_{t\to\infty} \theta_t,$$

and hence the Proposition 1 follows. Q.E.D.

Proof of Proposition 2. For the class of utility functions specified in (23) and (24), the equilibrium conditions (16) through (18) become, for every $t \geq 0$,

$$\lambda_t = \beta^t[1 + (1 - \sigma)\Phi]u_1(t), \tag{A.8}$$

$$\lambda_t F_2(t) = \beta^t(1 + \Phi)u_2(t), \tag{A.9}$$

$$\mu_t l_2(t) = \beta^t(1 + \Phi)u_2(t) + \frac{u_2(t)l_{22}(t)}{l_2^2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)]. \tag{A.10}$$

(i) Combining (A.8) and (A.9) we have

$$\frac{u_2(t)}{u_1(t)} = \frac{1 + \Phi}{1 + (1 - \sigma)\Phi} F_2(t).$$

Comparing it to (10) yields

$$\theta_t = \frac{\sigma\Phi}{1 + \Phi}, \quad \forall t \geq 0.$$

In order to prove (ii) and (iii), we need the following lemma which is proved after the proof of this position:

Lemma 3. Let

$$H_t = \frac{u_2(t)}{l_2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)].$$

Then, under the assumption of the Proposition 2, we have:

(a) $\phi_{-1} > 0 \Longrightarrow \phi_t > 0$ *and* $H_t < 0$ *for every* $t \geq 0$.

(b) $\phi_{-1} < 0 \Longrightarrow \phi_t < 0$ *and* $H_t > 0$ *for every* $t \geq 0$.

(c) $\phi_{-1} = 0 \Longrightarrow \phi_t = 0$ *and* $H_t = 0$ *for every* $t \geq 0$.

In the following proof of (ii) and (iii), we will always assume that $\Phi > 0$. (The degenerate case $\Phi = 0$ holds only if $g_t = 0$ for all $t \geq 0$, which is not interesting here.) In this case, $b_0^R + k_0 = 0$ (or < 0 or > 0) if and only if $\phi_{-1} = 0$ (or < 0 or > 0).

(ii) Since the optimal labor income tax rate θ_t is always at the steady state value, from the Proposition 1, the optimal capital income tax rate and the optimal level of investment tax credit reach the steady state value in period t for some $t \geq 0$ if and only if

$$F_2(t) = \frac{I_2(t)}{I_1(t)}, \quad F_1(t) = \frac{\delta'(h_t)}{I_1(t)}. \tag{A.11}$$

Necessary Condition: Multiplying both sides of Eq. (19) by $F_2(t)$ and using the condition $F_2(t) = I_2(t)/I_1(t)$ yields

$$\mu_t I_2(t) - \lambda_t F_2(t) = \frac{u_2(t) I_{12}(t)}{I_1(t) I_2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)]. \tag{A.12}$$

Substracting (A.9) from (A.10) yields

$$\mu_t I_2(t) - \lambda_t F_2(t) = \frac{u_2(t) I_{22}(t)}{I_2^2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)]. \tag{A.13}$$

Since $I_{22}(t) < 0$ and $I_{12}(t) > 0$ (from the concavity of I), (A.12) and (A.13) imply

$$H_t = \frac{u_2(t)}{I_2(t)}[\phi_t - \beta\phi_{t-1}(1 - \delta(h_t) + \delta'(h_t)h_t)] = 0. \tag{A.14}$$

From Lemma 3, then, (A.14) implies $\phi_{-1} = 0$.

Sufficient Condition: If $\phi_{-1} = 0$, then, by Lemma 3, $\phi_t = 0$ and $H_t = 0$ for every $t \geq 0$. Substituting them into the equilibrium conditions (17) through (20), then, yields Eq. (A.11) for every $t \geq 0$. That is, $\tau_t = \pi_t = \bar{\theta}$ for every $t \geq 0$.

(iii) If $\phi_{-1} > 0$, then, from Lemma 3, $\phi_t > 0$ and $H_t < 0$ for every $t \geq 0$. From (A.9) and (A.10), then, we have

$$\mu_t I_2(t) > \lambda_t F_2(t), \tag{A.15}$$

and from (19) and (20) we have

$$\lambda_t > \mu_t I_1(t), \tag{A.16}$$

$$\lambda_t F_1(t) > \mu_t \delta'(h_t). \tag{A.17}$$

Combining (A.15) with (A.16) and (A.17) respectively yields

$$\frac{I_2(t)}{I_1(t)} > F_2(t), \quad \frac{I_2(t) F_1(t)}{\delta'(h_t)} > F_2(t). \tag{A.18}$$

By comparing (A.18) with the first-order conditions (10) and (11), then, we have $\tau_t \geq \theta_t = \bar{\theta}$ and $\pi_t \geq \theta_t = \bar{\theta}$, and hence (iii) of the Proposition 2 for

$\phi_{-1} > 0$. Using exactly the same argument we can prove the case when $\phi_{-1} < 0$ as well. Q.E.D.

Proof of Lemma 3. First, substituting the equilibrium condition (14) into the definition of H_t we have

$$H_t = \frac{u_2(t)}{l_2(t)}\phi_t - \frac{u_2(t-1)}{l_2(t-1)}\phi_{t-1} \equiv \bar{\phi}_t - \bar{\phi}_{t-1}, \tag{A.19}$$

for $t \geq 1$.

(a) First we show that if $\phi_{-1} > 0$, then $H_0 < 0$ and $\phi_1 < 0$. Part (a) of the Lemma 3 can then be proved by induction.

If $\phi_{-1} > 0$ and $H_0 \geq 0$, then, by the definition of H_0 we have $\phi_0 > 0$ too. From (20) for $t = 1$, then, we have $\lambda_1 F_1(1) > \mu_1 \delta'(h_1)$. Substituting it into (21) yields

$$\mu_0 > \mu_1[1 - \delta(h_1) + \delta'(h_1)h_1]. \tag{A.20}$$

Since $H_0 \geq 0$, we have, from (A.10) and (14),

$$\begin{aligned}\mu_0 &\leq l_2^{-1}(0)(1+\Phi)u_2(0)\\ &= \beta(1+\Phi)l_2^{-1}(1)u_2(1)[1-\delta(h_1)+\delta'(h_1)h_1].\end{aligned} \tag{A.21}$$

Comparing (A.20) and (A.21) yields

$$\mu_1 \leq l_2^{-1}(1)\beta(1+\Phi)u_2(1),$$

which implies, from (A.10), $H_1 \geq 0$. So from $\phi_{-1} > 0$ and $H_0 \geq 0$ we have deduced $\phi_0 > 0$ and $H_1 \geq 0$. By induction, we have $H_t \geq 0$ for all $t \geq 0$. From (A.19), then, we have

$$\bar{\phi}_t \geq \bar{\phi}_{t-1} \geq \bar{\phi}_0 > 0. \tag{A.22}$$

But from the proof of Proposition 1 we know that if the economy converges to a steady state, then, $\lim_{t\to+\infty} \beta^{-t}\phi_t = 0$, which is inconsistent with (A.22). So, when $\phi_{-1} > 0$, it cannot be true that $H_0 \geq 0$. In other words, $\phi_{-1} > 0$ implies that $H_0 < 0$. Now we show that ϕ_0 cannot be nonpositive. If $\phi_0 \leq 0$, then, from (20) for $t = 1$, we have $\lambda_1 F_1(1) < \mu_1 \delta'(h_1)$. Using the similar argument we have just used above for the case of $H_0 \geq 0$, then, we can show that $H_1 < 0$ which implies $\bar{\phi}_1 < 0$. By induction, $H_t < 0$ for all $t \geq 0$. But that means $\bar{\phi}_{t+1} < \bar{\phi}_t < \bar{\phi}_1 < 0$ for all $t \geq 1$, which is again inconsistent with the steady state condition $\lim_{t\to+\infty} \beta^{-t}\phi_t = 0$. So, we have $\phi_0 > 0$.

(b) Similar to part (a).

(c) If $H_0 > 0$, then, $\phi_0 > 0$, and using the similar argument that we used in part (a) one can show that it implies that $H_t > 0$ for all t, which is not consistent with the steady state condition for ϕ_t. Similarly, $H_0 < 0$ would imply

$H_t < 0$ for all t, which again is not consistent with the steady state condition for ϕ_t. Thus, $H_0 = 0$, which implies that $\phi_0 = 0$. By induction, $\phi_t = 0$ and $H_t = 0$ for all $t \geq 0$. Q.E.D.

Proof of Proposition 3. Using the same argument as that used in the proof of Lemma 2, one can show that, for every $t \geq 1$, $\mu_t' = A_t \lambda_t'$ for the same A_t as that in Lemma 2. Substituting it into (31), one has that

$$\lambda_t' = [1 - \delta(h_{t+1}) + A_{t-1}^{-1} F_1(t+1) h_{t+1}] A_{t+1} A_t^{-1} \lambda_{t+1}', \quad \forall t \geq 1. \tag{A.23}$$

Comparing (A.23) to (A.2) yields

$$\frac{\lambda_{t+1}'}{\lambda_{t+1}} = \frac{\lambda_t'}{\lambda_t} = \gamma, \quad \forall t \geq 1,$$

for some constant γ. Using this equation, Eq. (30) and the fact that $\mu_t' = A_t \lambda_t'$, $t \geq 1$, then, we have, for every $t \geq 1$,

$$\mu_t' = \gamma \mu_t, \quad \lambda_t' = \gamma \lambda_t, \quad \phi_{t-1}' = \gamma \phi_{t-1}. \tag{A.24}$$

Substituting (A.24) into (26) through (32) and comparing them to (16) through (22), we have

$$\begin{aligned} &\gamma(1 + \Phi) u_1(t) + \gamma \Phi [u_{11}(t)(c_t - b_{0t}^c) - u_{12}(t)(n_t + e_t + b_{0t}^w)] \\ &= (1 + \Phi') u_1(t) + \Phi' [u_{11}(t)(c_t - b_{1t}^c) - u_{12}(t)(n_t + e_t + b_{1t}^w)], \quad \forall t \geq 1, \end{aligned} \tag{A.25}$$

and

$$\begin{aligned} &\gamma(1 + \Phi) u_2(t) + \gamma \Phi [u_{12}(t)(c_t - b_{0t}^c) - u_{22}(t)(n_t + e_t + b_{0t}^w)] \\ &= (1 + \Phi') u_2(t) + \Phi' [u_{12}(t)(c_t - b_{1t}^c) - u_{22}(t)(n_t + e_t + b_{1t}^w)], \quad \forall t \geq 1. \end{aligned} \tag{A.26}$$

Substituting (A.24) into (25) yields

$$\gamma \phi_0 = \beta^{-1} \Phi' (k_1 + b_1^R). \tag{A.27}$$

In other words, Proposition 3 holds if and only if there exist some restructured debt b_1^R and $\{b_{1t}^c, b_{1t}^w\}_{t \geq 1}$, a multiplier Φ', and a constant γ such that (A.25) through (A.27) and (33) hold for the Ramsey allocation. In the following, it will be shown that these conditions are satisfied by the Ramsey allocation when $u(.,.)$ is strictly concave.

Let

$$U'(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}, \quad U''(t) = \begin{pmatrix} u_{11}(t) & u_{12}(t) \\ u_{21}(t) & u_{22}(t) \end{pmatrix},$$

and let

$$z_t = \begin{pmatrix} c_t \\ -n_t - e_t \end{pmatrix}, \quad B_{it} = \begin{pmatrix} b_{it}^c \\ b_{it}^w \end{pmatrix}, \quad i = 0, 1.$$

Then, (A.25) and (A.26) can be written as

$$\begin{aligned} &\gamma(1+\Phi)U'(t) + \gamma\Phi U''(t)(z_t - B_{0t}) \\ &= (1+\Phi')U'(t) + \Phi' U''(t)(z_t - B_{1t}), \quad \forall t \geq 1, \end{aligned} \tag{A.28}$$

and, using (14), (33) can be written as

$$\sum_{t=1}^{\infty} \beta^{t-1}[U'(t)]^T (z_t - B_{1t}) = \frac{u_2(0)}{I_2(0)} \beta^{-1}(k_1 + b_1^R). \tag{A.29}$$

So, it will be sufficient to prove that there exist b_1^R, $\{b_{1t}^c, b_{1t}^w\}_{t\geq 1}$, Φ', and γ such that the conditions (A.27) through (A.29) are satisfied by the Ramsey allocation. When $u(.,.)$ is strictly concave, $U''(t)$ is an negative-definite matrix and is thus invertible. For given γ and Φ', then, if $\Phi' \neq 0$, (A.28) has a unique solution for B_{1t}. Let $B_t(\gamma, \Phi')$ denote the solution. When $\Phi' \neq 0$, (A.27) also has a unique solution for b_1^R, which will be denoted by $b_1^R(\gamma, \Phi')$. Substituting these solutions into (A.29) and multiplying it by Φ', one has that

$$\gamma\phi_0 \frac{u_2(0)}{I_2(0)} = \gamma(1+\Phi)\Lambda + \gamma\Phi \sum_{t=1}^{\infty} \beta^{t-1}[U'(t)]^T (z_t - B_{0t}) - (1+\Phi')\Lambda, \tag{A.30}$$

where

$$\Lambda = \sum_{t=1}^{\infty} \beta^{t-1}[U'(t)]^T [U''(t)]^{-1} U'(t) < 0.$$

Apparently, one can always choose an $\gamma \neq 0$ such that Eq. (A.30) has a nonzero solution for Φ'. Let γ^* be such a choice and let Φ^* be the corresponding solution to (A.30). Then, by construction, the solutions $B_t(\gamma^*, \Phi^*)$, $b_1^R(\gamma^*, \Phi^*)$, Φ^*, and γ^* along with the Ramsey allocation satisfy the conditions (A.27) through (A.29). Q.E.D.

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